Math 3371

Homework # 10

4.5 # 1, 2d, 7

5.2 # 2ab

4.5 #1 Let $f:[a,b] \to \mathbb{R}$ be continuous, and let L be between $f(a)$ & $f(b)$. We'll show $\exists c \in (a,b)$ with $f(c) = L$.

Pf By Thm 4.5.2, $f([a,b])$ is connected.

Since $f(a)$ and $f(b)$ are in $f([a,b])$, and L is between $f(a)$ & $f(b)$ and $f([a,b])$ is connected, L is also in $f([a,b])$, so $\exists c \in (a,b)$ with $f(c) = L$.

4.5 #2d This is impossible since $\mathbb{R}$ (the domain) is connected, but $\mathbb{Q}$ (the range) is disconnected. This violates Thm 4.5.2.

4.5 #7 Let $g(x) = f(x) - x$.

Then $g(0) = f(0) - 0 = f(0) \geq 0$ since f maps into $[0,1]$

$g(1) = f(1) - 1 \leq 0$ "

So by IVT $\exists y \in [0,1]$ s.t. $g(y) = 0$,

which means $f(y) - y = 0$,

i.e. $f(y) = y$ Done

5.2 #2 a

Let $f(x) = |x|$ Both are not differentiable
$g(x) = -|x|$. at 0.

But $fg(x) = |x||x| = x^2$ is differentiable at 0.

b $f(x) = |x|$ and $g(x) = 0$

then $fg(x) = 0$.