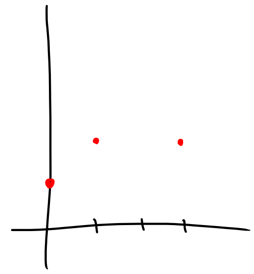
Math 3371 HW #11

Section 5.3 #3, #7

Section 6.2 #1a, 2a

5.3 #3bc $h(0)=1$ $h(1)=2$ $h(3)=2$



b By MVT using $a=0$, $b=3$

$\exists c \in (0,3)$ s.t.

$$f'(c) = \frac{f(3)-f(0)}{3-0} = \frac{1}{3}$$

c By MVT using $a=1$, $b=3$, $\exists$ some point d with $f'(d)=0$.

So $f'=0$ at one pt, and $f'=1/3$ at some other pt,

so by MVT (on f') there is some pt

with $f' = 1/4$.

5.3 #7 If $f'(x) \neq 1$. For a contradiction assume $f(a)=a$

and $f(b)=b$ for $a \neq b$.

Then MVT on $[a,b]$ gives $\exists c$ with

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{b-a}{b-a} = 1$$

which contradicts $f'(x) \neq 1$.

6.2 #1a $\qquad f_n(x) = \dfrac{nx}{1+nx^2}$

$f_n \to f(x) = \frac{1}{x}$ pointwise f. $x \in (0,\infty)$

Pf Let $x \in (0,\infty)$ and let $\varepsilon > 0$ be given. Will find $n \in \mathbb{N}$ s.t. $n > N \Rightarrow |f_n(x) - f(x)| < \varepsilon$

$$|f_n(x) - f(x)| = \left| \frac{nx}{1+nx^2} - \frac{1}{x} \right| = \dots = \left| \frac{-1}{(1+nx^2)x} \right|$$

$$\le \frac{1}{nx^3}.$$

Want $\frac{1}{nx^3} < \varepsilon$, so $n > \frac{1}{\varepsilon x^3}$

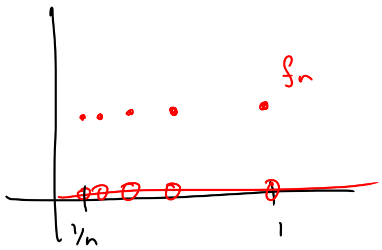
Let $N > \frac{1}{\varepsilon x^3}$. Then if $n > N$ we have

$$|f_n(x) - f(x)| \le \frac{1}{nx^3} < \frac{1}{\frac{1}{\varepsilon x^3} x^3} = \varepsilon$$ as desired.

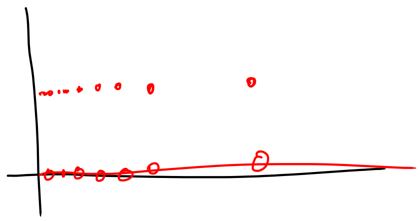
6.2 #2a

$$f_n(x) = \begin{cases} 1 & \text{if } x = 1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n} \\ 0 & \text{otherwise.} \end{cases}$$

$$f(x) = \begin{cases} 1 & \text{if } x = \frac{1}{k} \text{ for } k \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$



It converges to:



Each f_n is continuous at 0, but f is not.