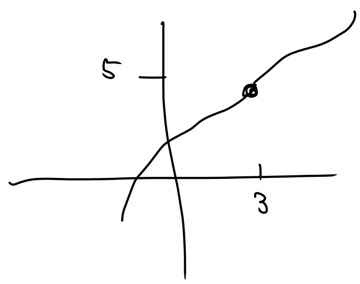
Math 3371
Real Analysis
Prof Chris Staecker

Real Analysis is about $\mathbb{R}$

mostly ideas from calculus.

The fundamental idea of calculus is the limit.

$$\lim_{x \to 3} f(x) = 5$$



Discussed in terms of "really really close"

When x is close to 3,

f(x) is close to 5.

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late 1600s Newton & Leibniz "invented calculus",

but never defined limit

$f'(x)$ $\frac{dy}{dx}$

1810s Bolzano published a real definition of limit, but nobody cared.

1820s Cauchy - same thing, nobody cared.

1850s Weierstrauss – same thing, but very well received.

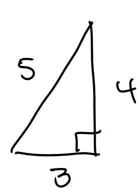
J. Grabiner: First the derivative was used, then discovered, explored, and developed, and only then defined.

Real #s & Rational #s

Pythagoreans Greeks ~ 500 BC

everything is geometric.

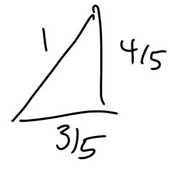
All measured in abstract units

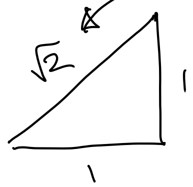


All numbers should be "commensurable"

↗

expressible as integers of some units.





Greeks proved $\sqrt{2}$ is not any ratio of the abstract unit.

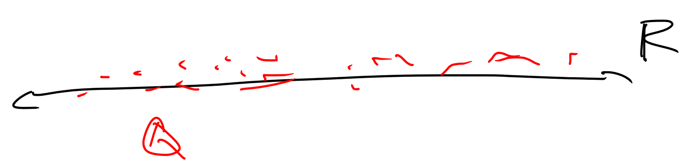
Greeks: not all lengths are numbers

~800s – 1000 AD   Arabs: these things $\sqrt{2}$ are also numbers, and we can do arithmetic with them.

$\mathbb{R}$ is a field, also ordered and "complete" ← no "gaps" or missing stuff in between real #s.

$\mathbb{Z}$ & $\mathbb{Q}$ are not complete.

integers   rationals



$\mathbb{R}$ is the only complete ordered field.

We'll talk a lot about distances between #s.

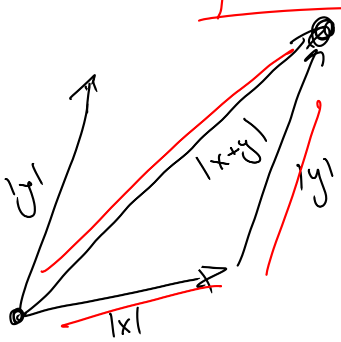
The distance from x to y is

$$d(x,y) = |y-x| = |x-y|$$

important fact:

$$\boxed{|x+y| \leq |x| + |y|}$$

The triangle inequality.



Also $x=y$ iff $|x-y| = 0$

Our first ε

Thm $x=0$ iff for any $\varepsilon>0$, $|x|<\varepsilon$.

$|x|$ is smaller than any positive #.

We imagine ε is super small.