Thm let $a_n \to a$ $\quad b_n \to b$, $\quad c \in \mathbb{R}$. Then

i. $\quad ca_n \to ca$ $\qquad \lim ca_n = ca$

ii. $\quad a_n + b_n \to a+b$

iii $\quad a_n b_n \to ab$

iv. $\quad \dfrac{a_n}{b_n} \to \dfrac{a}{b}$ if $b \neq 0$,

#i $\quad |ca_n - ca| = |c|\,|a_n - a|$ want $< \varepsilon$,

so make $|a_n - a| < \dfrac{\varepsilon}{|c|}$

$(a_n + b_n \to a+b)$ $\qquad \boxed{|x+y| \le |x| + |y|}$

#ii $\quad$ Let $\varepsilon > 0$ be given, we'll find N s.t.

$n > N \Rightarrow |a_n + b_n - (a+b)|$

$|a_n + b_n - (a+b)| = |a_n + b_n - a - b| = |a_n - a + b_n - b|$

$\le |a_n - a| + |b_n - b|$

we need $|a_n - a| < \varepsilon/2$

and $|b_n - b| < \varepsilon/2$.

Let N be so big that $n > N \Rightarrow$ $|a_n - a| < \varepsilon/2$ and $|b_n - b| < \varepsilon/2$.

Then if $n > N$ we have:

$|a_n + b_n - (a+b)| \le |a_n - a| + |b_n - b| < \varepsilon/2 + \varepsilon/2 = \varepsilon$

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