If A & B are open, then

$A \cup B$ is open

and $A \cap B$ is open.

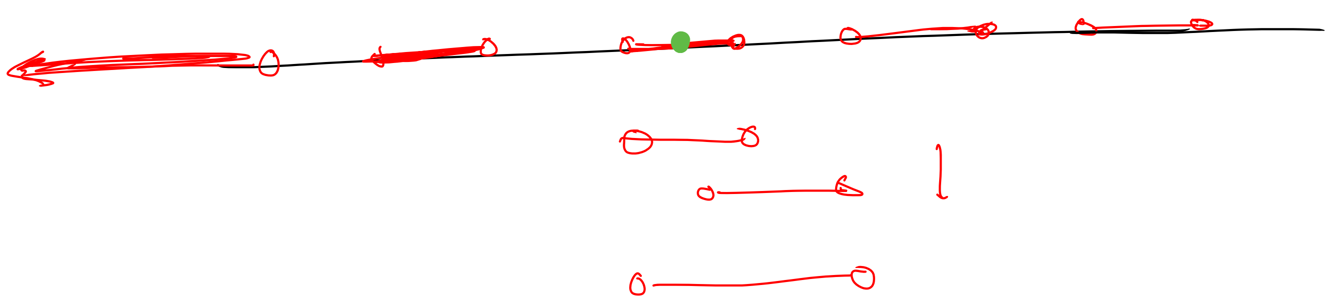
What about $A \cup B \cup C$?

$A \cup B$ is open, so

$$\underbrace{(A \cup B)}_{\text{open}} \cup \underbrace{C}_{\text{open}} \text{ is open.}$$

Thm Any finite union of open sets is open.

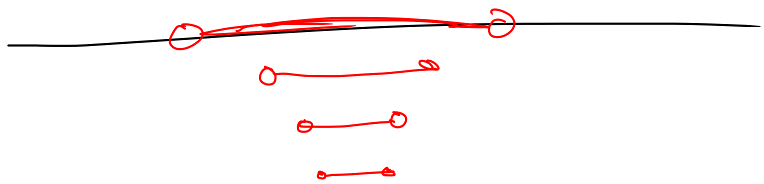
Also any finite intersection of open sets is open.



Thm Any union (even infinite) of open sets is open.

Is any (infinite) intersection of open sets still open?

$$\bigcap_{n=1}^{\infty} (-1/n, 1/n) = \{0\}$$ not open



Thm Any union of open sets is open.
Any finite intersection of open sets is open.

$$\bigcap (-1/n, 1+1/n) = [0, 1]$$

Closed

"An open set contains no boundary pts"

"A closed set contains all of its boundary pts"

Def A point x is a <u>limit point</u> <u>of a set A</u>

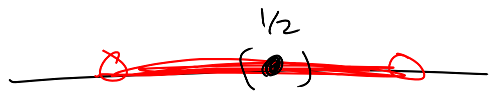
when: $\forall \varepsilon > 0$, $V_\varepsilon(x) \cap A$ contains points other than x.

$A = (0, 1)$

what are the limit points?

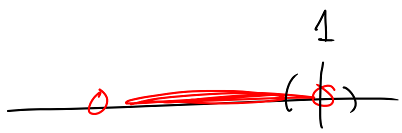
Is 2 a limit pt? NO — any $V_\varepsilon(2)$ has no intersection with A.

Is $1/2$ a limit pt?



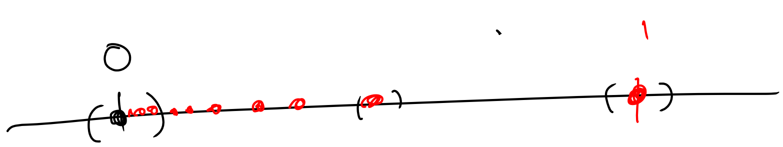
Yes! $V_\varepsilon(1/2)$ contains many points of A.

Is 1 a limit pt?



Yes! $V_\varepsilon(1)$ contains many pts of A.

$A = \{ 1/n \mid n \in \mathbb{N} \}$



$\frac{1}{1000} - \frac{1}{1001}$

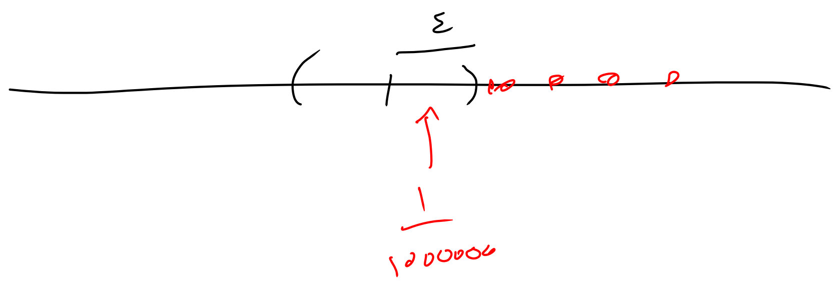
1 is not a limit pt: $V_\varepsilon(1) \cap A = \{1\}$

0 is a limit pt.

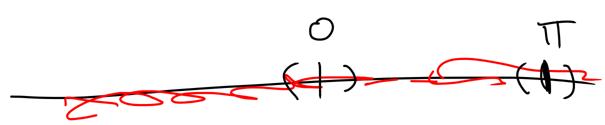
for $A = \{1/n\}$, 0 is the only limit pt.

$\varepsilon = .000000001$

$$\frac{1}{10000000001}$$



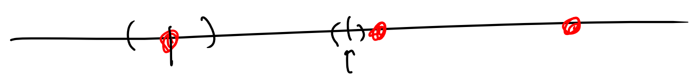
$A = \mathbb{Q}$



0 is a limit pt. also π, etc.

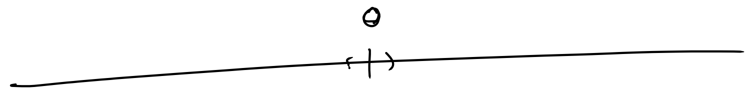
The set of limit pts is $\mathbb{R}$

$A = \{1, 2, 3\}$



has no limit points.

$A = \emptyset$



There are no limit pts.

Thm x is a limit pt of A

iff

$\exists$ a sequence $(a_n) \in A$, $a_n \neq x$ $\forall n$.

with $a_n \to x$.

Def F is a closed set if

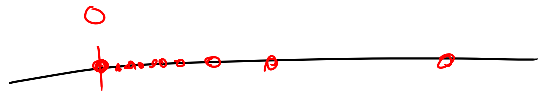
F contains all limit pts of F.

$F = (0,1)$ is not closed, because 0 is a limit pt, but $0 \notin F$.

$F = [0,1]$ is closed.

$F = \{1/n\}$ is not closed since 0 is a limit pt and $0 \notin F$.

$F = \{1/n\} \cup \{0\}$



this is a closed set.

$F = \{1, 2, 3\}$ has no limit pts, so automatically it is closed.

Any finite set is closed.

$F = \emptyset$ is closed. (its closed & open CLOPEN)

$F = \mathbb{Q}$ is not closed.

<u>Thm</u> The interval $[a, b]$ is closed.

<u>Pf</u> Let x be a limit pt of $[a,b]$.
We'll show $x \in [a,b]$.

Since x is a limit pt, $\exists$ a sequence
$a_n \to x$ $\quad \underline{a_n \in [a,b]} \ \forall n$.

means $a \le a_n \le b$,

So $a \le x \le b$ by order limit thms

So $x \in [a,b]$ Shewn.