A limit point of a set A is some pt x such that any nbhd $V_\varepsilon(x)$ has some pts of A other than x.

$V_\varepsilon(x) \cap A$ contains pts other than x.

A is closed when A contains all its limit pts.

$[a, b]$ is closed.

$\{1, 2, 3\}$ is closed (has no limit pts)

Every set has a closure

Def Let A be a set, and let L = set of limit pts of A.

The closure of A is

$$\overline{A} = A \cup L.$$

$\overline{A}$ is closed.

Thm $\bar{A}$ is the smallest closed set containing A.

Pf Let F be a closed set containing A, then we'll show $\bar{A} \subseteq F$.

$\bar{A} = A \cup L$ ← set of limit pts

So to show $\bar{A} \subseteq F$, enough to show $A \subseteq F$ and $L \subseteq F$.

$A \subseteq F$ since F contains A.
Since F is closed, it contains all limit pts of F,
so contains all limit pts of A.

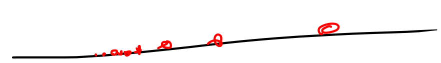
So $L \subseteq F$.

$\overline{(a,b)} = [a,b]$

$\overline{\{1,2,3\}} = \{1,2,3\}$

$\overline{\mathbb{N}} = \mathbb{N}$

$\overline{\{1/n \mid n \in \mathbb{N}\}}$



$\overline{\mathbb{Q}} = \mathbb{R}$

$$\overline{\{\tfrac{1}{n} \mid n \in \mathbb{N}\}} = \{1/n \mid n \in \mathbb{N}\} \cup \{0\}$$

$$\frac{1}{n} + (-1)^n$$

$$(1, 1/2, 1, 1/2, 1, 1/2, 1, 1/2, \ldots)$$

has subseqs converging to

1 or $1/2$

$$\underline{1, 1/2}, \underline{1, 1, 1/2, 1/2}, \underline{1, 1, 1, 1/2, 1/2, 1/2}, \ldots$$