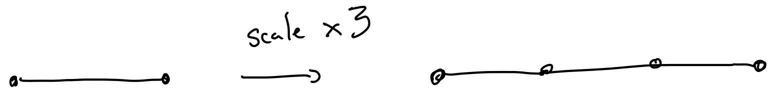
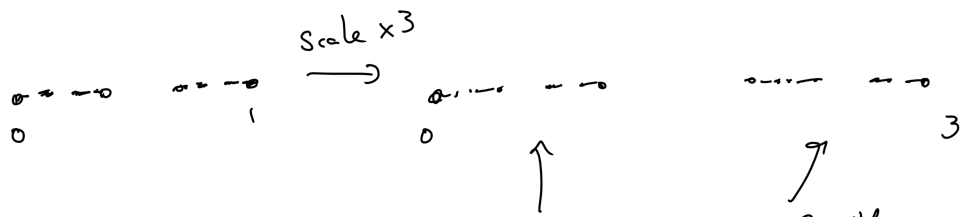
Fractals

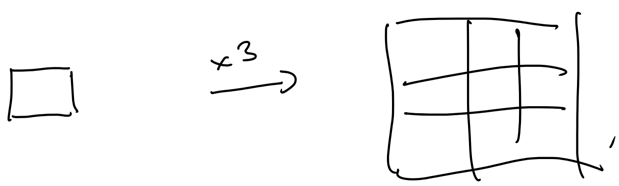
[0,1] 

3 copies of the original.

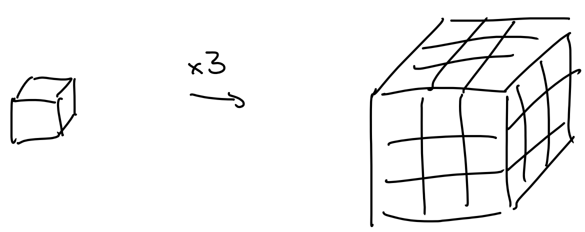
C: 

2 copies of the original

3^{1} copies.



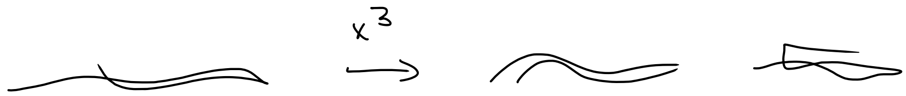
$9 = 3^{2}$ copies.



$27 = 3^{3}$ copies

This exponent is the fractal dimension
or Hausdorff dimension

Cantor set



2 copies.

$3^{?} \leftarrow$ dim.

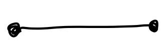
The fractal dimension of Cantor set

is $\log_2 3 = .631\ldots$

Fractals can have non-integer dimension.

List of fractals by Hausdorff dimension

Compact Sets



closed intervals are nice,

many theorems in calc. are about closed intervals.

<u>Thm</u> If $f : [a,b] \to \mathbb{R}$ is continuous

and $f(a) < 0 < f(b)$, then

$\exists\, c \in [a,b]$ with $f(c) = 0$.

or NIP

Is this really about closed intervals?

Closed intervals are:

- closed sets
- connected
- compact

<u>Def</u> A set $K \subseteq \mathbb{R}$ is <u>compact</u> when:

if $(a_n) \in K$ is a sequence of points of K, then it has a convergent subseq, and the limit is in K.

<u>Ex</u> A closed interval $[c,d]$ is compact.

Let $(a_n) \in [c,d]$,

(a_n) is <u>bounded</u>, so it has a conv. subseq by BW.

Since $[c,d]$ is a <u>closed set</u>, the limit is contained in $[c,d]$.

<u>Thm</u> Any bounded closed set is compact.

So Cantor set is compact,
$\{1, 2, 3\}$ is compact.

$(0,1)$ is not compact:

$(1/n) \in (0,1)$, but the limit is not in $(0,1)$.

Any non-closed set is not compact.

<u>Thm Any compact set is closed</u>

$\mathbb{R}$ is not compact

(n) is a sequence with no convergent subseq.

Same for any unbounded set

An unbounded set is not compact

Any compact set is bounded.

Thm A set is compact
iff it is closed and bounded.