$$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Thm If f is differentiable at c,
then f is continuous at c.

Pf We assume $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists and equals $f'(c)$

So $$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

So $$\lim_{x \to c} f(x) - f(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \cdot (x - c)$$

$$= \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \cdot \lim_{x \to c} (x - c)$$

$(\lim x) + k$

$\lim (x + k)$

$$= f'(c) \cdot 0 = 0$$

So $\lim_{x \to c} f(x) - f(c) = 0$ so $\lim_{x \to c} f(x) = f(c)$

so f is continuous at c.

Shown.

Derivative Rules

- $(f+g)'(x) = f'(x) + g'(x)$
- $(kf)'(x) = k \cdot f'(x)$

$\Big\}$ the derivative is Linear

- $(fg)'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$ (product rule)
- $(f/g)'(x) = \dfrac{g(x)\, f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$ (quotient rule)

$\lim_{x \to c} \dfrac{f(x) - f(c)}{x-c}$

Prove the product rule

$$(fg)'(c) = \lim_{x \to c} \frac{f(x) \cdot g(x) - f(c) \cdot g(c)}{x-c}$$

$$= \lim_{x \to c} \frac{f(x) g(x) - f(x) g(c) + f(x) g(c) - f(c) g(c)}{x-c}$$

$$= \lim_{x \to c} \frac{f(x) g(x) - f(x) g(c)}{x-c} + \frac{f(x) g(c) - f(c) g(c)}{x-c}$$

$$= \lim_{x \to c} f(x) \cdot \frac{g(x) - g(c)}{x-c} + \lim_{x \to c} g(c) \frac{f(x) - f(c)}{x-c}$$

$$= f(c) \cdot g'(c) + g(c) f'(c)$$

The Chain Rule

$$(g\circ f)'(x) = g'(f(x))\cdot f'(x)$$

Fake proof:

$$(g\circ f)'(c) = \lim_{x\to c}\frac{g(f(x))-g(f(c))}{x-c}$$

$$= \boxed{\lim_{x\to c}\frac{g(f(x))-g(f(c))}{f(x)-f(c)}}\cdot\frac{f(x)-f(c)}{x-c}$$

$$= \boxed{g'(f(c))}\quad f'(c)$$

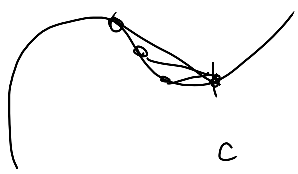
bogus

For Reals: Let f & g be differentiable.
(f is diff. at c & g is diff. at $f(c)$)

Then $(g\circ f)'(c) = g'(f(c))\cdot f'(c)$.

Pf Since g is differentiable:

$$g'(f(c)) = \lim_{y\to f(c)}\boxed{\frac{g(y)-g(f(c))}{y-f(c)}}$$



call this $d(y)$

c

so $d(y) = \begin{cases} \dfrac{g(y) - g(f(c))}{y - f(c)} & \text{if } y \neq f(c) \\ g'(f(c)) & \text{if } y = f(c) \end{cases}$

This $d(y)$ is continuous for any y, and:

$$d(y)\,(y - f(c)) = g(y) - g(f(c))$$

true for any $y \in \mathbb{R}$.

Now let $y = f(x)$, then

$$d(f(x))\,(f(x) - f(c)) = g(f(x)) - g(f(c))$$

so $$d(f(x))\,\frac{f(x) - f(c)}{x - c} = \frac{g(f(x)) - g(f(c))}{x - c}$$

Now do $\lim_{x \to c}$ on both sides

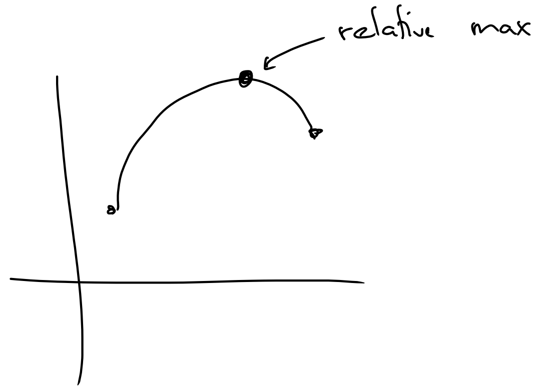
$ab = 4$

$a\frac{b}{2} = 2$

$$d(f(c)) \cdot f'(c) = (g \circ f)'(c)$$

$$\parallel$$

$$g'(f(c)) \cdot f'(c) = (g \circ f)'(c)$$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