Math 1015: Exam 1

Question 1. (12 points) Please explain why Condorcet's method satisfies the monotonicity criterion.

If we boost a Condorcet winner, then they will still win all pairwise comparisons (perhaps by even more votes), so they will still be the winner.

Question 2. (14 points) Please compute the Shapley-Shubik power index for $[12 : 8, 5, 3]$

A B C

perms	weights	pivotal
A B C	8 5 3	B
A C B	8 3 5	B
B A C	5 3 8	A
B C A	5 8 3	A
C A B	3 8 5	B
C B A	3 5 8	A

A: 3/6

B: 3/6

C: 0/6

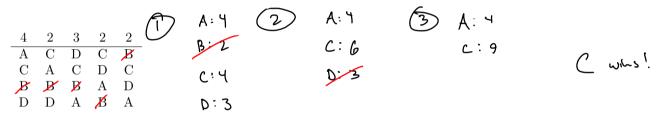**Question 3.** This whole page is about this election:

4	2	3	2	2
A	C	D	C	B
C	A	C	D	C
B	B	B	A	D
D	D	A	B	A

a) (6 points) Please find the winner using plurality, or say if there is a tie or no winner.

A: 4
B: 2
C: 4
D: 3

A & C tie

b) (8 points) Please find the winner using Instant Runoff Voting, or say if there is a tie or no winner.

4	2	3	2	2
A	C	D	C	~~B~~
C	A	C	D	C
~~B~~	~~B~~	~~B~~	A	D
D	D	A	~~B~~	A

① A: 4
   ~~B: 2~~
   C: 4
   D: 3

② A: 4
   C: 6
   ~~D: 3~~

③ A: 4
   C: 9

C wins!

c) (8 points) Please find the winner using Condorcet's method, or say if there is a tie or no winner.

A: 8 vs B: 5 → A vs B : A   B vs C : C

A vs C : C   B vs D : B

A vs D : D   C vs D : C

C is the Condorcet winner!

Question 4. (12 points) Please write a few sentences about the Shapley-Shubik & Banzhaf power indexes: what are they useful for in the real world? Why do we need them? (Don't compare and contrast the two, just say generally why we might need either one of them. You don't need to write a lot.)
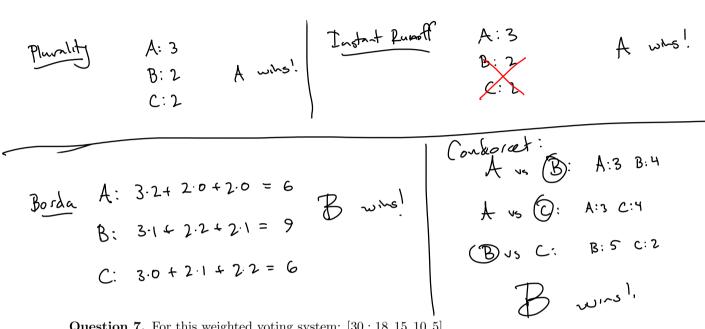
We need these because the weights can be misleading, like in [9:8,8,1] it looks like the 1 has less power, but actually it has power equal to the others. SS & Banzhaf tell the real power better than the weights do.

Question 5. (12 points) For this plurality election here:

4	2	3
A	C	D
C	A	C
B	B	B
D	D	A

explain how the voters in the third column could manipulate the outcome. Include some words of explanation— don't just write a bunch of numbers.

The winner was A, which is the worst result for the 3rd column.

If they change to
C
D
A
B ,
then the winner becomes C, which is a better outcome.

Question 6. (12 points) For this election, find the winner 3 times using 3 different methods (your choice). Clearly label your three different methods.

3	2	2
A	B	C
B	C	B
C	A	A

Plurality

A: 3
B: 2
C: 2

A wins!

Instant Runoff

A: 3
B̶:̶ ̶2̶
C̶:̶ ̶2̶

A wins!

Borda

A: $3 \cdot 2 + 2 \cdot 0 + 2 \cdot 0 = 6$

B: $3 \cdot 1 + 2 \cdot 2 + 2 \cdot 1 = 9$

C: $3 \cdot 0 + 2 \cdot 1 + 2 \cdot 2 = 6$

B wins!

Condorcet:

A vs (B): A:3 B:4

A vs (C): A:3 C:4

(B) vs C: B:5 C:2

B wins!

Question 7. For this weighted voting system: $[30 : 18, 15, 10, 5]$

a) (4 points) Is the 18 a dictator? Please explain why or why not.

NO – 18 does not meet the quota by itself.

b) (6 points) Is the 5 a dummy? Please explain why or why not.

No – I can do $15 + 10 + 5 = 30$, and here the 5 is important in meeting the quota.

c) (6 points) Do any voters have veto power? Please explain why or why not.

No – even the 18 is not needed to reach 30, since we can do $15 + 10 + 5 = 30$.