

Name: _____

Math 1015: Exam 1

Question 1. (12 points) Please write a few sentences about the Shapley-Shubik & Banzhaf power indexes: what are they useful for in the real world? Why do we need them? (Don't compare and contrast the two, just say generally why we might need either one of them.)

We need these because the weights can be misleading, like in $[9:8,8,1]$ it looks like the 1 has less power, but actually it has power equal to the others. SS & Banzhaf tell the real power better than the weights do.

Question 2. (14 points) Please compute the Banzhaf power index for $[12:8,5,3]$

<u>combs</u>	<u>total</u>	<u>A</u>	<u>B</u>	<u>C</u>
A B C	16	x	x	
A B	13	x	x	
A C	11			
B C	8			
A	8			
B	5			
C	3			
\emptyset	0			

A: 2/4
B: 2/4
C: 0/4

Question 3. This whole page is about this election:

2	4	2	3
C	D	C	B
A	C	D	C
B	B	A	D
D	A	B	A

a) (6 points) Please find the winner using plurality, or say if there is a tie or no winner.

A: 0 C: 4
B: 3 D: 4 C & D tie

b) (8 points) Please find the winner using Instant Runoff Voting, or say if there is a tie or no winner.

2	4	2	3
C	D	C	B
A	C	D	C
B	B	A	D
D	A	D	A

Rd 1
~~A: 0~~
 B: 3
 C: 4
 D: 4

Rd 2
~~B: 3~~
 C: 4
 D: 4

Rd 3
 C: 7
 D: 4 C wins!

c) (8 points) Please find the winner using Condorcet's method, or say if there is a tie or no winner.

A vs B: A: 4 B: 7

A vs C: A: 0 C: 11

A vs D: A: 2 D: 9

B vs C: B: 5 C: 6

B vs D: B: 5 D: 6

C vs D: C: 7 D: 4

C is the Condorcet winner.

Question 4. (12 points) For this election, find the winner 3 times using 3 different methods (your choice). Clearly label your three different methods.

3	2	2
A	B	C
B	C	B
C	A	A

Plurality

A: 3

B: 2

C: 2

A wins!

Instant Runoff

A: 3

~~B: 2~~

~~C: 2~~

A wins!

Borda

	3	2	2
2	A	B	C
1	B	C	B
0	C	A	A

$$A: 3 \cdot 2 + 2 \cdot 0 + 2 \cdot 0 = 6$$

$$B: 3 \cdot 1 + 2 \cdot 2 + 2 \cdot 1 = 9$$

$$C: 3 \cdot 0 + 2 \cdot 1 + 2 \cdot 2 = 6$$

B wins!

Condorcet:

A vs B: A: 3 B: 4

A vs C: A: 3 C: 4

B vs C: B: 5 C: 2

B wins!

Question 5. (12 points) Please explain why the Borda count method satisfies the unanimity criterion.

If all voters rank X above Y , then
 X gets more points than Y , so Y won't win.

Question 6. For this weighted voting system: $[30 : 18, 15, 10, 5]$

a) (4 points) Is the 18 a dictator? Please explain why or why not.

NO - 18 does not meet the quota by itself.

b) (6 points) Is the 5 a dummy? Please explain why or why not.

No - I can do $15 + 10 + 5 = 30$, and here
the 5 is important in meeting the quota.

c) (6 points) Do any voters have veto power? Please explain why or why not.

No - even the 18 is not needed to reach 30,
since we can do $15 + 10 + 5 = 30$.

Question 7. For this plurality election here:

4	2	3
A	C	D
C	A	C
B	B	B
D	D	A

A wins!

- a) (4 points) Please explain one specific way to change the votes which would be regarded as an “irrelevant” change according to the IIA criterion.

If I change

A		A
C	to	B
B		C
D		D

, this is irrelevant.

- b) (4 points) Please explain one specific way to change the votes which would not be regarded as an “irrelevant” change according to the IIA criterion.

If I change

A		C
C	to	A
B		B
D		D

this is not irrelevant.

- c) (4 points) Does your example in either part a or part b above demonstrate that plurality does not satisfy IIA? Please explain why or why not.

For mine, no. In part a, after the change, A still wins, so IIA is not violated.