Math 1015: Homework #8

Question 1. a) Draw an example of a graph having 8 vertices and 13 edges.

b) Next to each vertex, write its degree.

c) Say if your graph is connected or not.

Question 2. Draw an example of a graph with 6 vertices having degrees 2, 2, 2, 3, 3, and 4.

Question 3. My daughter plays little league softball in the town of Fairfield (true story). There are 8 teams in her age-group, with 12 girls on each team (made-up numbers). Imagine a graph where each vertex is a girl on some team, and two vertices are connected by an edge if those two girls are on the same team.

a) What is the degree of my daughter's vertex in this graph?

b) Is the graph connected? Say why or why not.

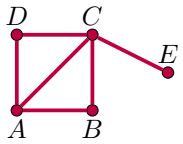
Question 4. Imagine a graph where each vertex is a word in the "Happy birthday to you" song, ignoring the person's name. (There should be 5 vertices.) Two vertices are connected by an edge when the two words have some letter in common.

a) Draw this graph.

b) Redraw the graph, but with your own name added into the song.

c) Instead of your name, think of a name that we could add that makes the graph become disconnected. (You don't have to use a real name— you can just make up some jumble of letters that would work.)

Question 5. Please write the formal definition of this graph as sets of vertices and edges:



Question 6. a) Please draw a picture of the graph formally defined by:

$$\text{vertices} : \{A, B, C, D, E\}, \qquad \text{edges} : \{(A, D), (A, C), (C, D), (B, E)\}$$

b) Is this graph connected? Explain specifically according to the definition of connectedness. (You should say something about paths.)

Question 7. Consider the graph in Question 5.

a) Does this graph have an Euler circuit? Either draw an Euler circuit, or say why none exists.

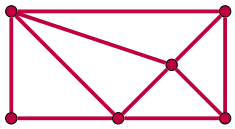
b) Does this graph have an Euler path? Either draw an Euler path, or say why none exists.

c) Use this graph as a specific example to demonstrate Euler's sum-of-degrees theorem.

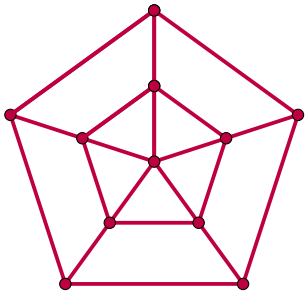
Question 8. Imagine I have 5 friends, and some of them are friends with each other too. My friends are Tony, Erin, Chuck, Patty, and Shawn. (Yes these are real people.) Tony is friends with 3 of these 5 people, Erin is friends with 2 of them, Chuck is friends with 1 of them, and Patty is friends with 3 of them. Shawn is not friends with any of them. Explain using Euler's sum-of-degrees theorem why this is impossible.

Question 9. For each of these graphs, say whether or not it has an Euler circuit, and whether or not it has an Euler path.

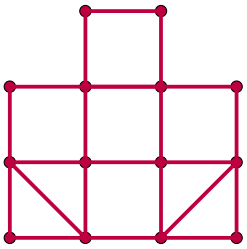
a)



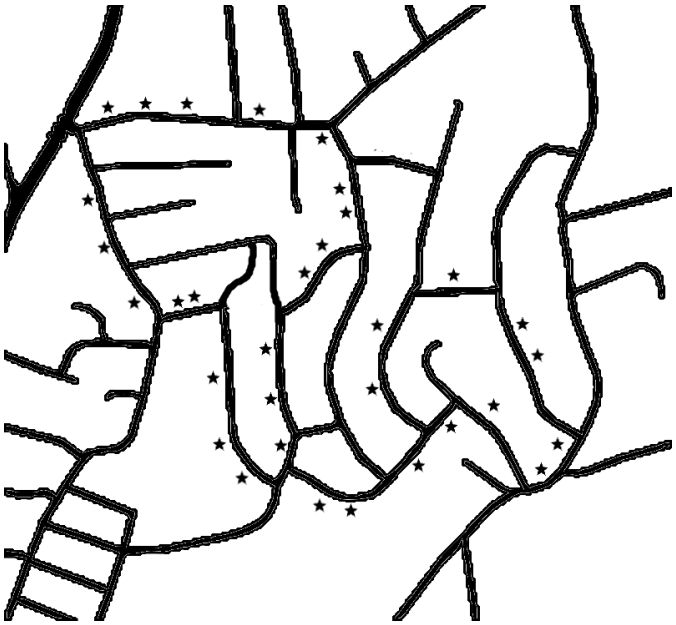
b)



c)



Question 10. My daughter needs to deliver her Girl Scout Cookies in this neighborhood:



The lines are streets, and each star is a house where she needs to bring her cookies.

a) Translate this picture into a graph, including only the streets that my daughter has deliveries on. (Most streets will not be included on your graph.)

b) Does this graph has an Euler circuit? Does it have an Euler path?

c) Give some real-world interpretation of your answer in part b in terms my 10 year old daughter would understand. If you like, begin your answer with "OK, honey,"