

## IIA criterion

If we change votes without changing anyone's ranking relative to the winner, then the results won't change.

like if B was the winner, and I change



IIA is not satisfied by Borda.

On HW, you'll show Plurality & IRV fail the IIA criterion.

and dictatorship does satisfy IIA.

|              | Maj | CWC | Mon | Unanimity | IIA |
|--------------|-----|-----|-----|-----------|-----|
| Plurality    | ✓   | ✗   | ✓   | ✓         | ✗   |
| Cond.        | ✓   | ✓   | ✓   | ✓         | ✓   |
| Borda        | ✗   | ✗   | ✓   | ✓         | ✗   |
| IRV          | ✓   | ✗   | ✗   | ✓         | ✗   |
| Dictatorship | ✗   | ✗   | ✓   | ✓         | ✓   |

In Condorcet method, an "irrelevant" change cannot effect the pairwise matchups involving the winner, so the result won't change.

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In the 1950 K. Arrow asked: can we design a system which satisfies the all (and always chooses a winner) ?

### Arrow's Impossibility Theorem

The only voting system possible which always chooses a winner and satisfies IIA & Unanimity is dictatorship.

This means there cannot be a "perfect" or "best" voting system.

I don't mean we haven't found a perfect one yet!

I mean it's impossible

## Manipulability

Can the voters cause weird effects by voting in weird ways?

Def A voting system is manipulable if voters can achieve a better result (in their own opinion) by voting differently from their true preferences.

This is generally bad: we want voters to be able vote how they feel.

Plurality System is manipulable.

look at B/G/N.

| Z.g. | Z.g. | 1 | B wins with plurality |
|------|------|---|-----------------------|
| B    | G    | N |                       |
| G    | B    | G |                       |
| N    | N    | B |                       |

These ones, to get a better result,  
should have voted G > B.

The Nader voters would've gotten a better outcome if they voted contrary to  
their true opinion.

So Plurality is manipulable.

"tactical voting"

What about IRV?

|   |   |   |
|---|---|---|
| 6 | 4 | 3 |
| A | C | B |
| B | B | C |
| C | A | A |

IRV:

I: A: 6

~~B: 3~~  
C: 4

II: A: 6

~~C: 7~~

C wins!

change one     $\begin{matrix} A \\ B \\ C \end{matrix}$     to     $\begin{matrix} B \\ A \\ C \end{matrix}$

|   |   |   |   |
|---|---|---|---|
| 5 | 1 | 4 | 3 |
| A | B | C | B |
| B | A | B | C |
| C | C | A | A |

IRV: I:

~~A: 5~~  
~~B: 4~~  
~~C: 4~~

A wins

So IRV is manipulable

## The Gibbard - Satterthwaite Theorem

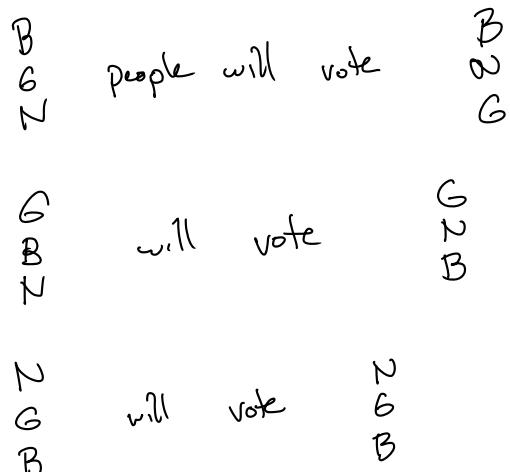
The only non-manipulable system is dictatorship!

Manipulability is extreme with Borda:

Bush / Gore / Nader with Borda:

|             |                 |                 |               |
|-------------|-----------------|-----------------|---------------|
| real votes: | $\frac{2.7}{B}$ | $\frac{2.7}{G}$ | $\frac{1}{2}$ |
|             | G               | B               | 0             |
|             | Z               | Z               | B             |

one common tactic : dumping



What will the results be?

|   |                 |                 |               |
|---|-----------------|-----------------|---------------|
| 2 | $\frac{2.7}{B}$ | $\frac{2.7}{G}$ | $\frac{1}{2}$ |
| 1 | B               | 0               | 0             |
| 0 | G               | Z               | B             |

$$B: 2.9 \cdot 2 + 2.9 \cdot 0 + .1 \cdot 0 = 5.8$$

$$G: 2.9 \cdot 0 + 2.9 \cdot 2 + .1 \cdot 1 = 5.9$$

$$N: 2.9 \cdot 1 + 2.9 \cdot 1 + .1 \cdot 2 = 6$$

N wins!

Borda is "dangerous" because tactical voting can cause very weird outcomes.