IIA criterion

If we change votes without changing anyone's ranking relative to the winner, then the results won't change.

like if B was the winner, and I change

A
B
D
C

→

A
B
C
D

This ought to be "irrelevant."

IIA is not satisfied by Borda.

On HW, you'll show Plurality & IRV fail the IIA criterion.

and dictatorship does satisfy IIA.

	Maj	CWC	Mono	Unanimity	IIA
Plurality	✓	X	✓	✓	X
Cond.	✓	✓	✓	✓	✓
Borda	X	X	✓	✓	X
IRV	✓	X	X	✓	X
Dictatorship	X	X	✓	✓	✓

In Condorcet method, an "irrelevant" change cannot effect the pairwise matchups involving the winner, so the result won't change.

In the 1950 K. Arrow asked: can we design a system which satisfies them all (and always chooses a winner)?

Arrow's Impossibility Theorem

The only voting system possible which always chooses a winner and satisfies IIA & Unanimity is dictatorship.

This means there cannot be a "perfect" or "best" voting system.

[I don't mean we haven't found a perfect one yet!

I mean it's impossible]

Manipulability

Can the voters cause weird effects by voting in weird ways?

Def A voting system is <u>manipulable</u> if voters can acheive a better result (in their own opinion) by voting differently from their true preferences.

This is generally bad: we want voters to be able vote how they feel.

Plurality System is manipulable.

look at B/G/N:

2.9.	2.9	.1
B	G	N
G	B	G
N	N	B

B wins with plurality

These ones, to get a better result, should have voted G N B.

The Nader voters would've gotten a better outcome if they voted contrary to their true opinion.

"tactical voting"

So Plurality is manipulable.

What about IRV?

6	4	3
A	C	B
B	B	C
C	A	A

IRV: I: A: 6, ~~B: 3~~, C: 4

II: A: 6, C: 7

C wins!

change one $\begin{matrix} A \\ B \\ C \end{matrix}$ to $\begin{matrix} B \\ A \\ C \end{matrix}$

5	1	4	3
A	B	C	B
B	A	B	C
C	C	A	A

IRV: I: A: 5, ~~B: 4~~, ~~C: 4~~

A wins

So IRV is manipulable

The Gibbard - Satterthwaite Theorem

The only non-manipulable system is dictatorship!

Manipulability is extreme with Borda:

Bush / Gore / Nader with Borda:

real votes:

2.9	2.9	.1
B	G	N
G	B	G
N	N	B

one common tactic: dumping

B	people will vote	B
G		N
N		G

G	will vote	G
B		N
N		B

N	will vote	N
G		G
B		B

What will the results be?

	2.9	2.9	.1
2	B	G	N
1	N	N	G
0	G	B	B

B: $2.9 \cdot 2 + 2 \cdot 9 \cdot 0 + .1 \cdot 0 = 5.8$

G: $2.9 \cdot 0 + 2 \cdot 9 \cdot 2 + .1 \cdot 1 = 5.9$

N: $2.9 \cdot 1 + 2.9 \cdot 1 + .1 \cdot 2 = 6$

N wins!

Borda is "dangerous" because tactical voting can cause very weird outcomes.