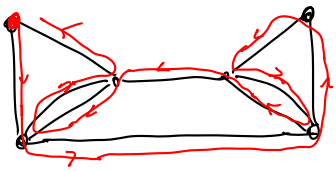
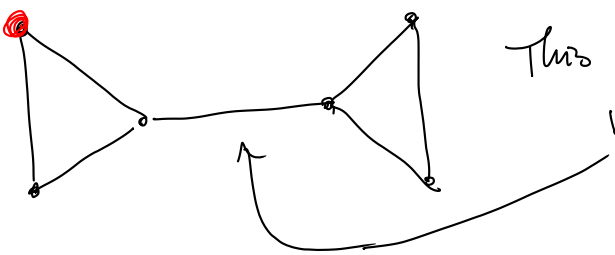
Euler Circuits

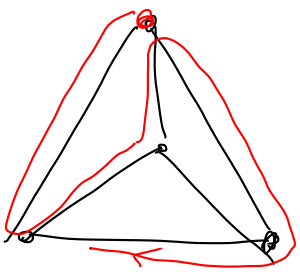
A path starting & ending at the same vertex which uses each edge exactly once.



This has an E. circ.



This has no E. circ, because of this "bridge"



Also has no E. circ.

Euler figured out how to tell if an E. circ. exists.

"if and only if" or "iff"

We say "A iff B" when A & B are logically equivalent.

A iff B means:

if A, then B

and if B, then A.

e.g. I am a parent iff I have a child

Are these correct?

I am old enough to drink iff I am 35 years old. ✗

I have seen Lucas the Stag iff I have been to F.U. ✗

x is even iff x is divisible by 2 ✓

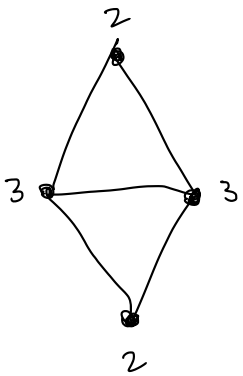
x is Marc Nemec iff x is pres. of FU.

✓ assuming only 1 MN exists.

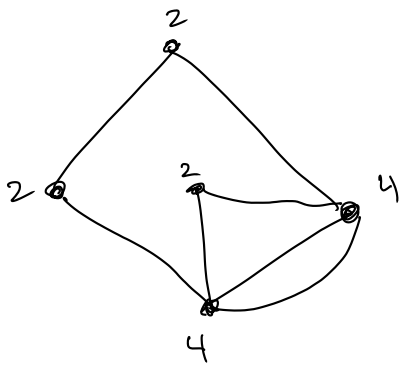
x is even iff x+1 is odd ✓

Euler's Theorem abt E. circuits

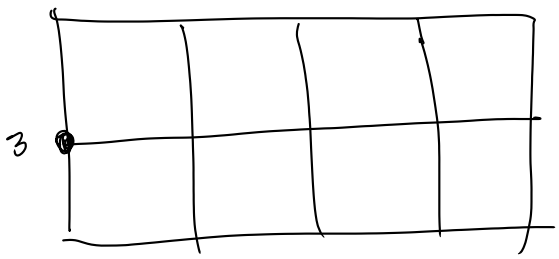
A graph has an E. circuit
iff all verts have even degree.



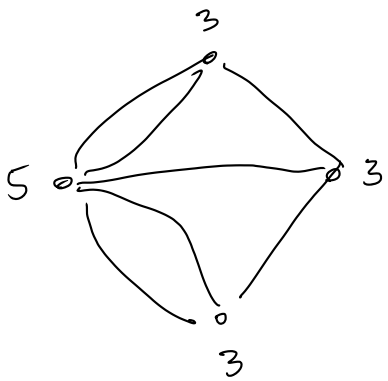
No E. circ.



does have an E. circ.



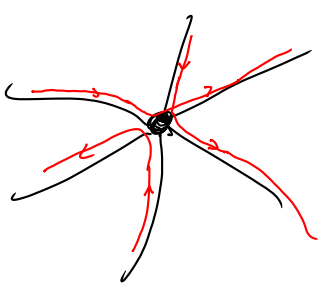
No E. circ.



No E. circ.

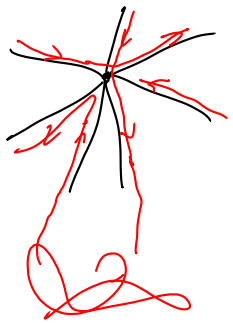
This is an "existence Theorem"

Why it's true:



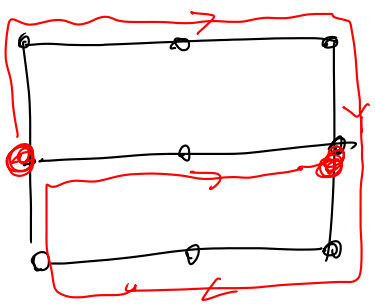
We need to arrive & leave along different edges.

if the # of edges is odd, we'll get stuck.

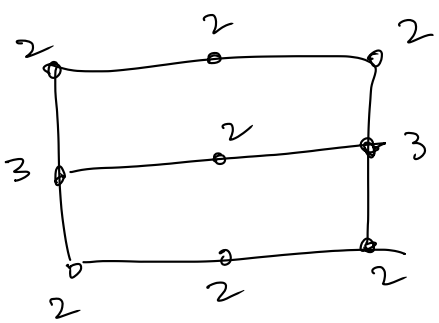


if degree is odd, the circuit gets stuck.

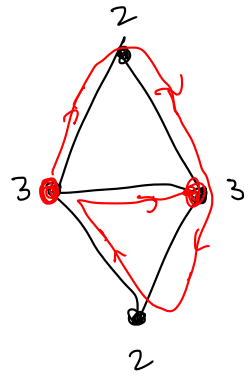
Variation: What about a path using each edge once, (but not a circuit)



This is an Euler path



Thm An E. path exists when all verts have even degree except 2 odds.
(The 2 odds are the start & end of the path)

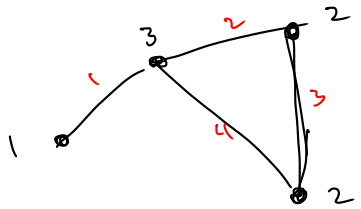


Has 2 odds, so an E. path exists.

So an E. circ exists if there's no odds

. - - - - path . . - - - - - -- 2 odds.

What if there's only 1 vertex with odd degree?



If we add up the degrees,

we get $2 \times$ (# of edges)

Thm (Sum-of-degrees theorem)

The sum of degrees equals $2 \times$ (# of edges)

(therefore sum of degrees is always even)

Example about NFL

2 conferences, 13 teams in each

NFL decided each team plays 14 games,

11 in-conference, 3 outside of conference.

This is impossible: consider a graph of 13 teams & 11 in conference games each.

each vert has 11 edges

each degree is 11.

total sum of degrees is $11 \times 13 = 143$

odd is impossible.