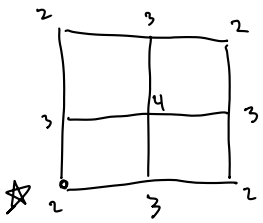
Often, no Euler path / circuit exists

The best we can do is a <u>minimum duplication circuit</u>

min. dup. circ. means we use each edge, but repeat as few edges as possible.

How do we find a min. dup. circ?

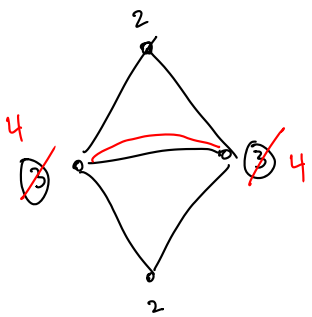


Mr Plow wants to cover each edge without repeats.

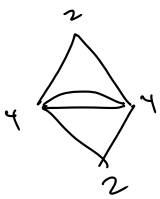
What's the best we can do?

(can we do it with only 1 repeat?)

Strategy: "Eulerize the graph by <u>edge doubling</u>"

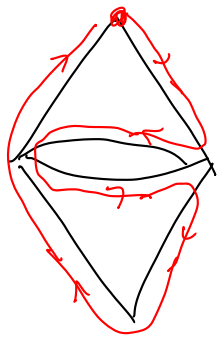


Make all degrees even by doubling existing edges.



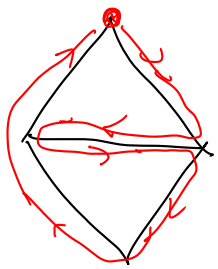
This is the "Eulerized" graph.

We can now find an Euler circuit.

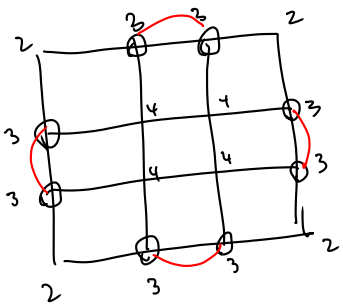


on the original graph, this is a circuit with 1 edge repetition.

OG:

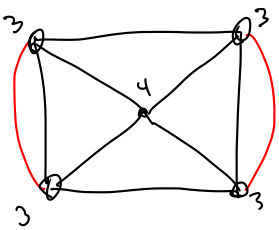


This is a min. dup. circ, duplicating 1 edge.

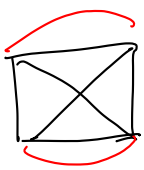


Let's Eulerize!

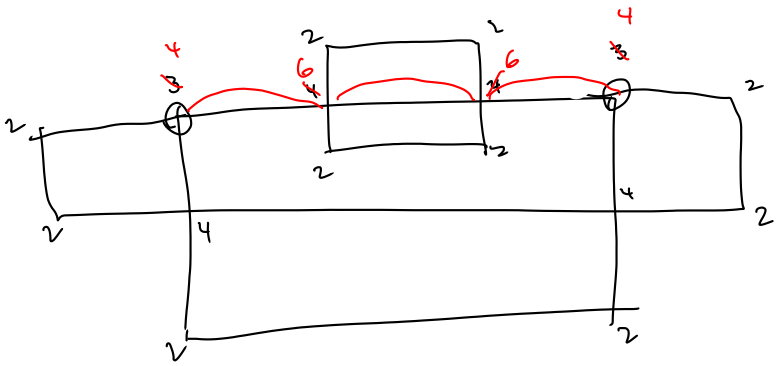
Pair up the odds & connect them by doubling edges.



OR



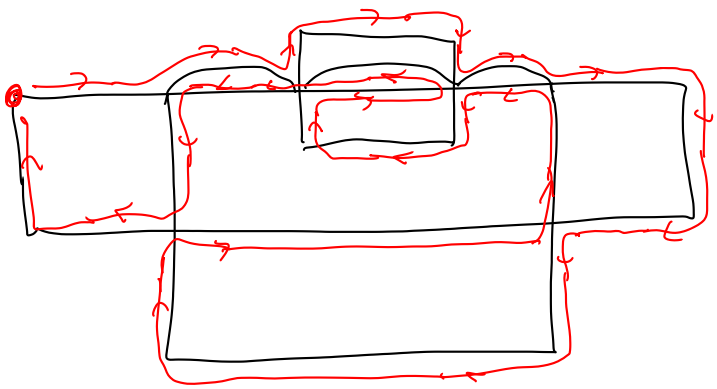
Sometimes they can't pair up:



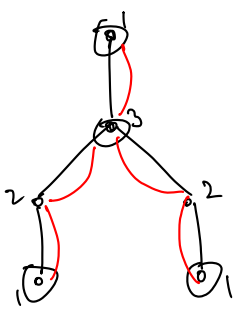
can't invent a new edge connecting them.

If we have odds with no edge existing between them, double each edge along a path.

Here, we'd get



The min. dup. circ requires 3 edge dupes.



here we need 5 dupes.

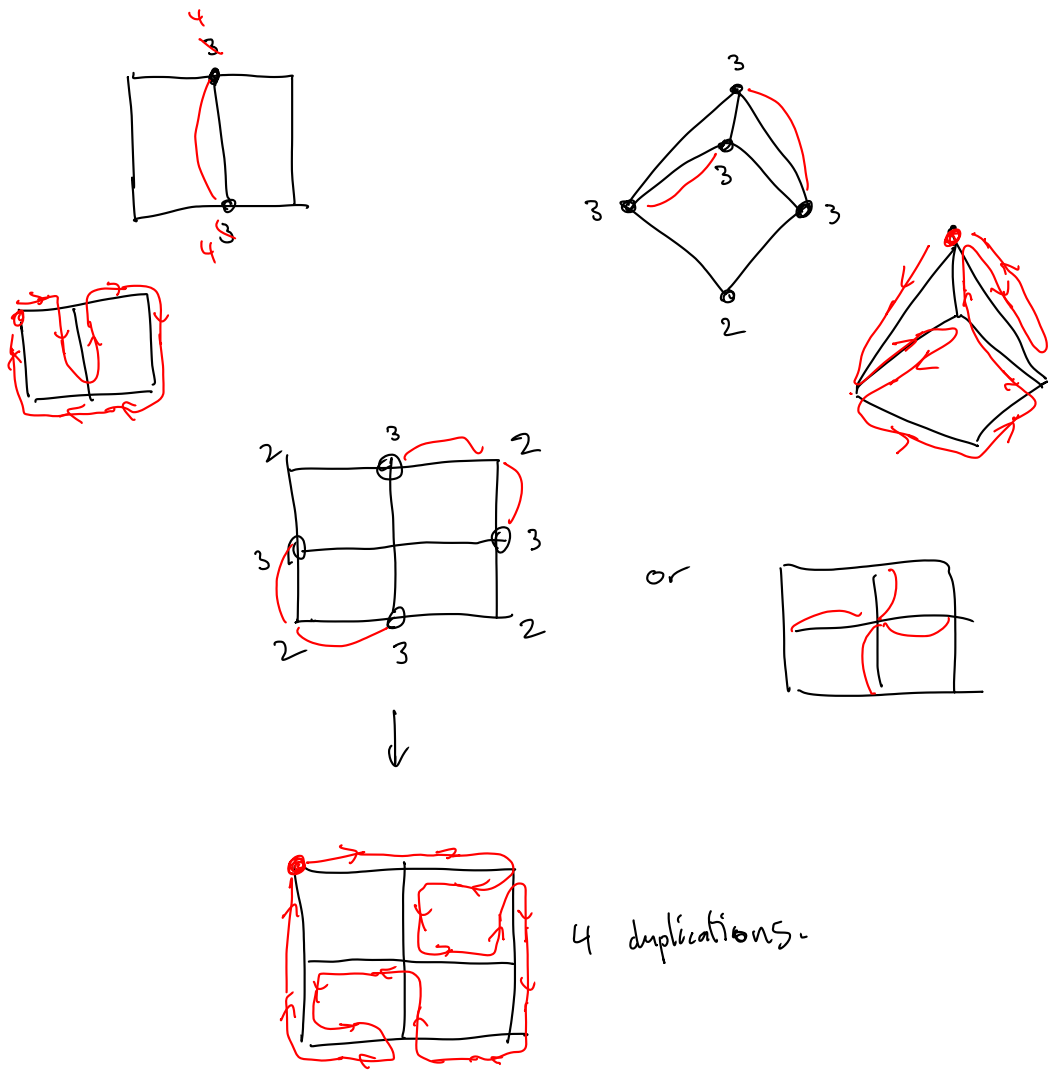
Min Dup Circ. process:

Eulerize by edge doubling:

- connect up odds by doubling edges or paths.

Make an E. circuit on the Eulerized,

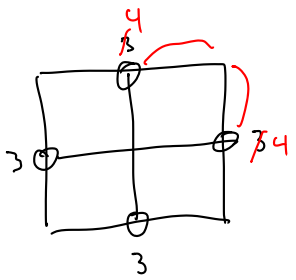
this makes a min. dup. cic. on the original graph.



Min. Dup. Path!

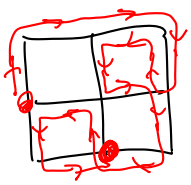
Covers all edges with as few repeats as possible, but starts & ends at different points.

Same process, but when Eulerizing, leave 2 odds, make the rest even.

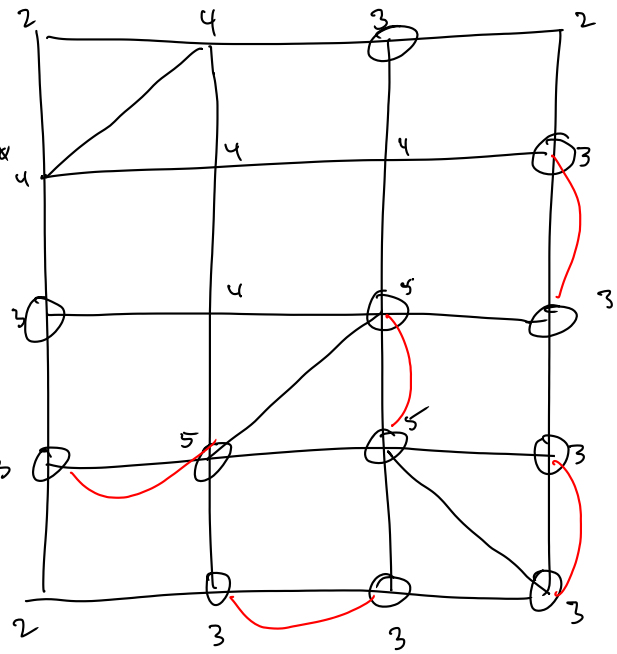


find a min dup path.

Now our path starts & ends at the odds.



How many edge dupes would be required for a min. dup. path?



We need 5 edge dupes!