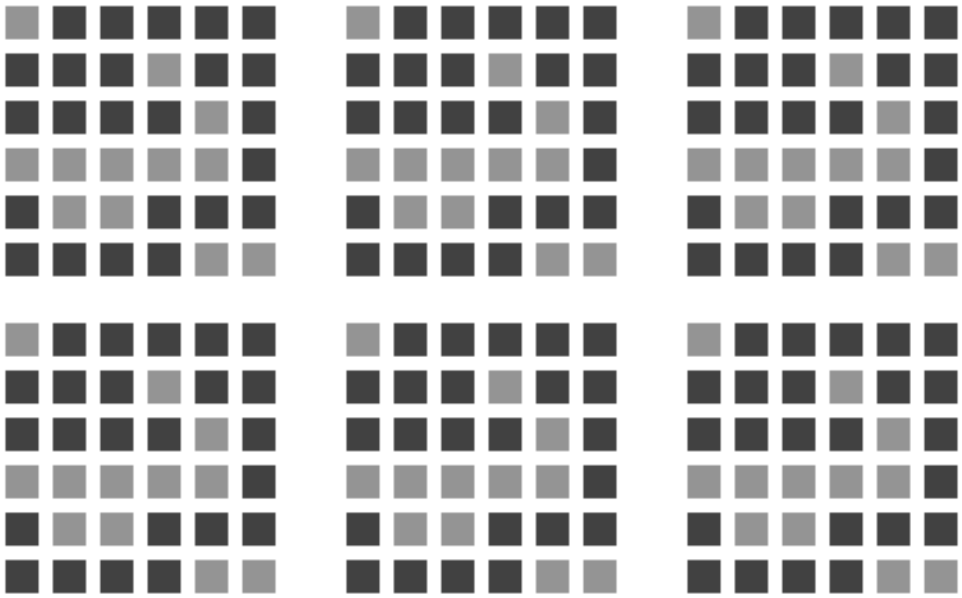
Name: _____

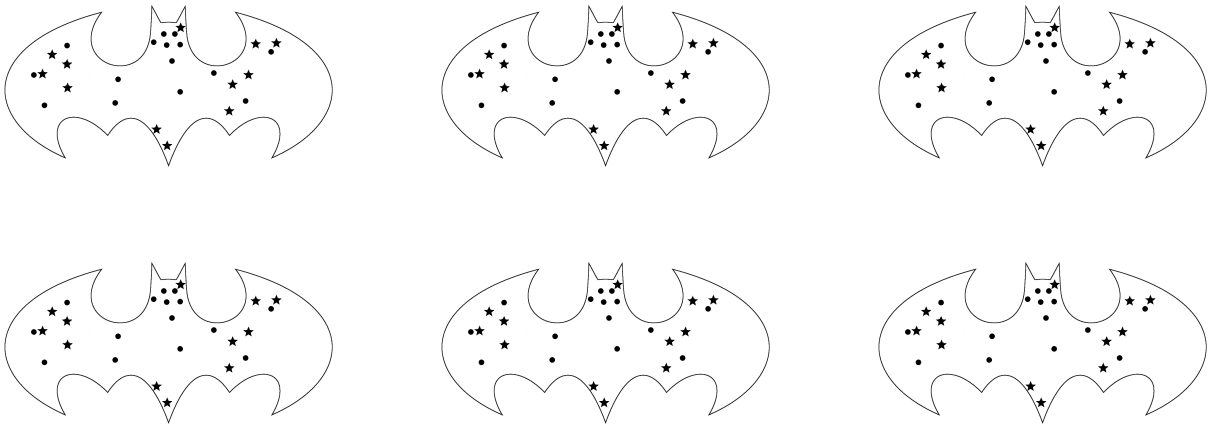
Exam #2 topics and sample questions

Gerrymandering

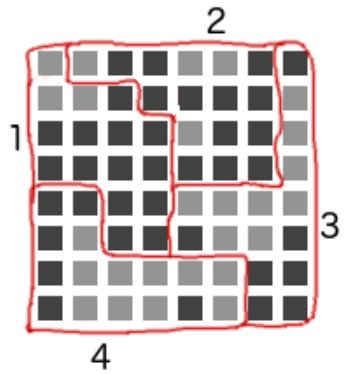
1. In these grids, there are 36 voters, 24 of whom are dark, and 12 are light. We want to divide them into 4 districts of 9 voters each. Draw districts in different ways to try to make all possible outcomes in terms of the number of districts which are majority dark vs majority light. If any outcomes are impossible, explain why.



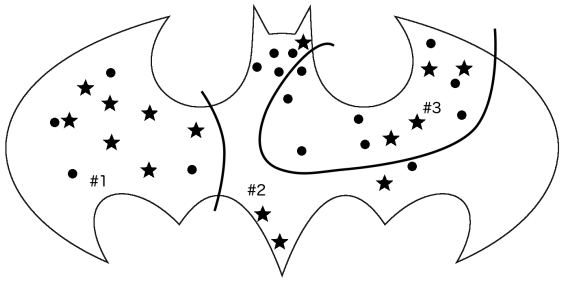
2. Same question, but for these pictures. Divide it into 3 districts of 9 voters each.



3. For these districts, compute the efficiency gap.



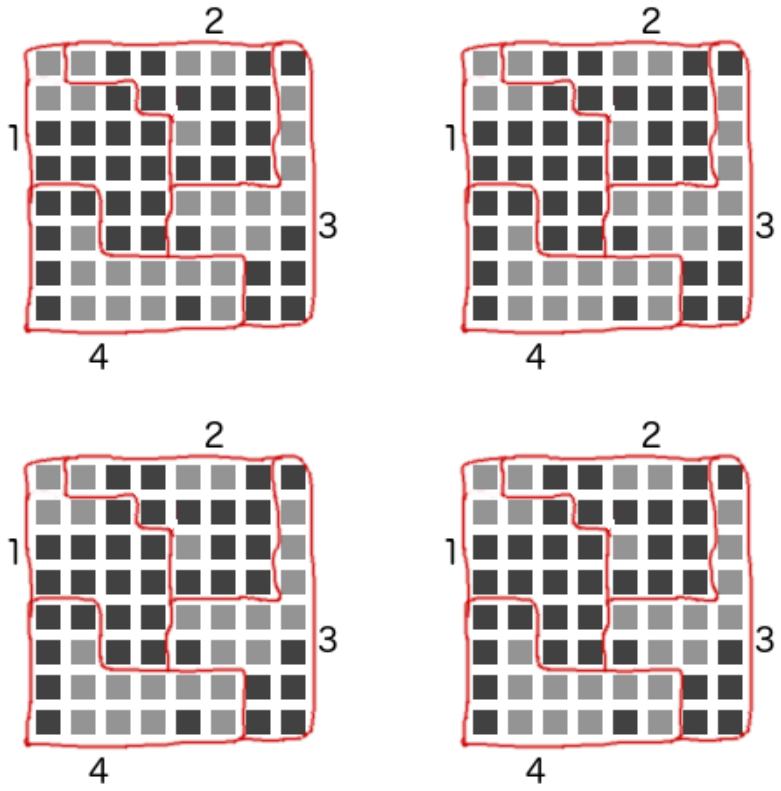
4. For these districts, compute the efficiency gap.



5. These are the shapes of the 5 (real) Connecticut house of representatives districts.

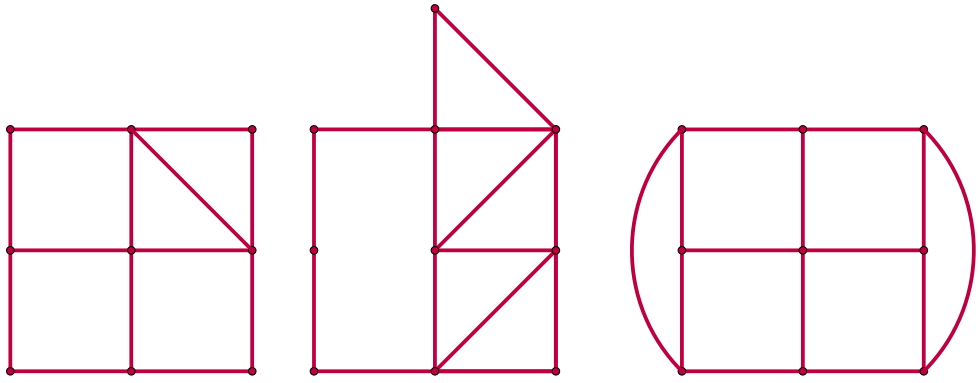
On top of each picture, draw the outline of the convex hull.

6. Find the convex hull ratio and isoperimetric quotient of each district:



Graph theory

7. Draw an example of a connected graph with 4 vertices having degrees 1,1,3, and 3.
8. Explain why it's impossible to have a graph with 6 vertices having degrees 1, 2, 2, 2, 3, 3.
9. Imagine a graph with a vertex for each word in the english language, and an edge connecting two words if they rhyme. Is this graph connected? Which word has greater degree: "red", or "orange"?
10. For each of these graphs, decide if it has an Euler circuit or an Euler path, or neither.



Answers!

1. We need 5 voters to win each district, so it is not possible for the lights to win more than 2, since they only have 12. It is possible for the darks to win them all. So the possible outcomes are: 4D/0L, 3D/1L, 2D/2L.

2. There are 12 stars and 15 circles. We need 5 voters to win a district, so it is possible for circles to win all 3, but stars can only win at most 2. So the possible outcomes are: 3 circle/0 star, 2 circle/1 star, 1 circle/2 star.

3. The efficiency gap is $\frac{12}{64}$ in favor of the lights.

4. The efficiency gap is $\frac{4}{32}$ in favor of the circles.

6. #1 has CH= $\frac{16}{19.5}$ and IQ= $\frac{4\pi \cdot 16}{20^2}$
   #2 has CH= $\frac{16}{21.5}$ and IQ= $\frac{4\pi \cdot 16}{20^2}$
   #3 has CH= $\frac{16}{24}$ and IQ= $\frac{4\pi \cdot 16}{24^2}$
   #4 has CH= $\frac{16}{20}$ and IQ= $\frac{4\pi \cdot 16}{20^2}$

8. This is impossible because the sum of degrees in any graph must be even.

9. It is not connected, "red" has greater degree.

10. The left-most one has an Euler path, the middle one has an Euler circuit, and the one on the right has neither.