Name: _____

Math 3342 Exam #2

Question 1. (7 points each) For each language, please make a grammar:

a) $\{(ab)^n a^n\}$

$S \to abSa \mid \varepsilon$

b) $\{a^n b^k \mid k = n \text{ or } k = n+2\}$

$S \to aSb \mid b^2 \mid \varepsilon$

c) $\{a^n b^k c^m\}$

$S \to ABC$
$A \to aA \mid \varepsilon$
$B \to bB \mid \varepsilon$
$C \to cC \mid \varepsilon$

d) $\{a^{2n} b a^k b^n\}$

$S \to aaSb \mid bT$
$T \to aT \mid \varepsilon$

e) Which of your grammars above are context-free, and which are not? Briefly explain (enough so that I know that you know what "context free" means).

All are context-free since the left side of every production is a single non-terminal symbol.

Question 2. (10 points) Please make a grammar for the language of strings that look like:

`[XXX] [XXXX] [XX] [] [X]`

These are strings consisting of zero or more blocks, each of which has brackets around 0 or more X's. (The empty string is part of this language.)

$S \to [T]S \mid \varepsilon$

$T \to XT \mid \varepsilon$

Question 3. (10 points) Please give an example of a nonempty set L such that some derivative of L equals L. Write clearly which derivative of L you are using.

$L = \{a^n\}$

then $\frac{d}{da}L = \{a^{n-1}\} = \{a^n\}$

so $L = \frac{d}{da}L$

Question 4. (5 points each) Here is a stack machine:

	read	pop	push
1.	a	S	SXX
2.	b	X	ε
3.	ϵ	S	ε

a) Please show that *aabbbb* is accepted by this stack machine.

$(aabbbb, S) \mapsto (abbbb, SXX) \mapsto (bbbb, SXXXX) \mapsto (bbbb, XXXX)$
$\mapsto (bbb, XXX) \mapsto (bb, XX) \mapsto (b, X) \mapsto (\varepsilon, \varepsilon)$ ✓

b) Please show that *ab* is not accepted.

$(ab, S) \mapsto_1 (b, SXX) \mapsto_3 (b, XX) \mapsto (\varepsilon, X)$ stuck!

or $(ab, S) \mapsto_3 (ab, \varepsilon)$ stuck!

c) Please describe the language of this machine in words or using set theory notation.

$\{a^n b^{2n}\}$

d) If we change the "push ε" in rule 3 to "push X", then what will the language of this new machine be?

$\{a^n b^{2n+1}\}$

e) If we change the "push ε" in rule 2 to "push X" (but keep rule 3 like it originally was), then what will the language of this new machine be?

Now it is impossible to clear X from the stack, so only ε will be accepted.

$L = \{\varepsilon\}$

Question 5. (10 points) Please show that $L = \{a^k b^n a^n\}$ is not a regular language.

Let $D_i = \frac{\partial}{\partial b^i} L = \{b^{n-i} a^n\}$

(This derivative kills all strings for which $k \neq 0$.
When $k=0$, we cancel some b's)

Then strings in D_i have i fewer b's than a's, so D_i is different for each i, so L has ∞-ly many different derivatives, so L is not regular.

Question 6. (10 points) Please explain briefly why there can be no general purpose method to convert any context-free grammar into an equivalent NFA.

Many CFGs will generate a non-regular language
[e.g. $S \to aSb \mid \varepsilon$ which is $\{a^n b^n\}$.]

But a NFA must always generate a regular language.

So there can be no general method to convert any CFG to NFA.