DFAs:

F G C H

Farmer has a goat, cabbage, hyena

Farmer & one other can cross over at a time,

G&C cannot be alone,

G&H ------

What sequence of moves gets them all to the other side?

A DFA with alphabet: $\Sigma = \{H, G, C, F\}$

H means take the Hyena across,

etc.

F means F goes across alone.

so HGGCFH means ...

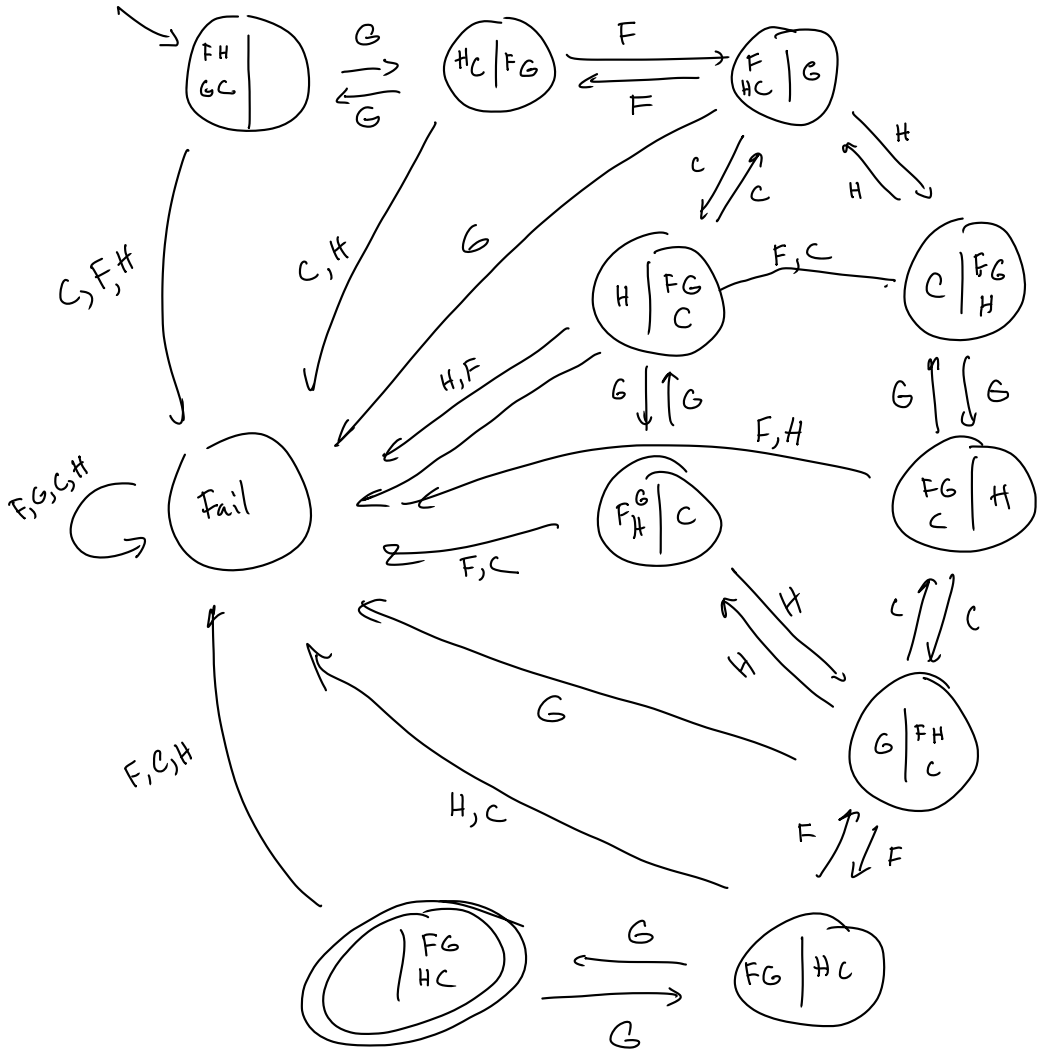
not a valid solution

Our DFA will accept only valid solutions.

Q's to answer: What is the solution?

Is it unique?

How many solutions?




Two "good" solutions (shortest length),

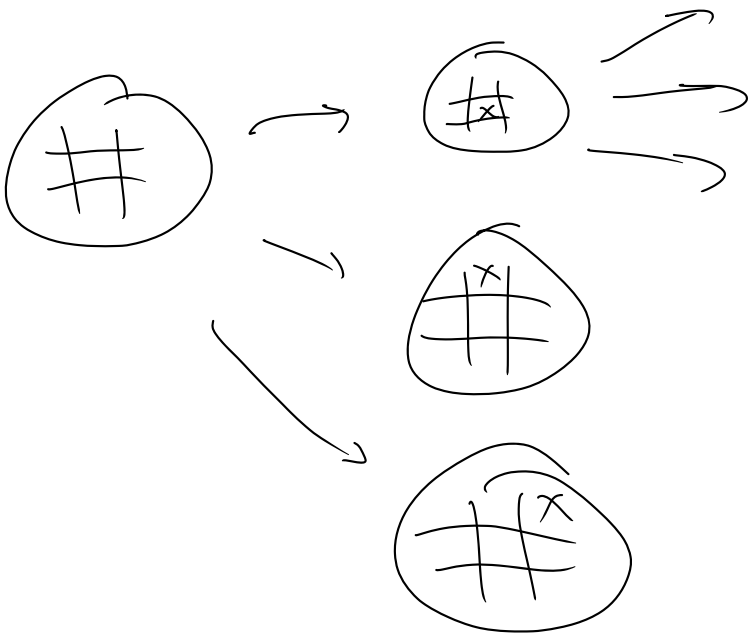
∞-ly many other solutions going around loops.

GFCGHFG

GFHGCFG

Could make a DFA for

winning at tic-tac-toe



Or Rubik's cube

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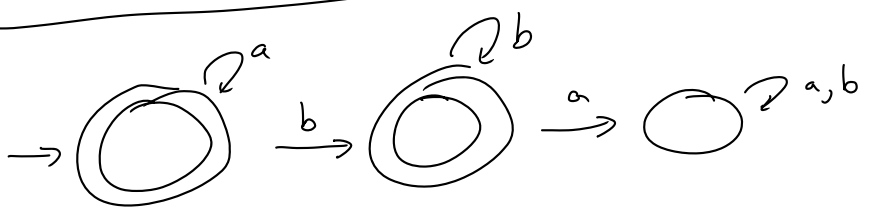



what strings are accepted?

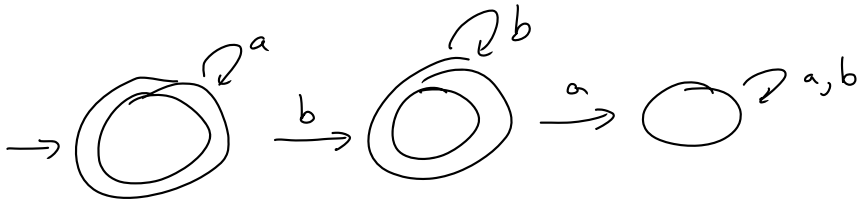
ε, a, b, aaaaa, bbbbb

not accepted: ba, aba

Accepts when all a's are before all b's

Accepts the set:

$$\{a^n b^m \mid n, m \geq 0\}$$




This set is the language accepted by the DFA.

For a DFA $M$, the language is written $L(M)$.

Big question: Which sets can be the language of a DFA?

$\{a^n \mid n \text{ is prime}\}$ actually has no DFA

Def A language is called regular if it is the language of some DFA.

Q: Which languages are regular?

To show a language is regular, you must build a DFA to accept it.

To show a language is not regular, we must prove no possible DFA can accept it.

(much harder)

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## A fancy DFA for binary numbers

$$\Sigma = \{0, 1\}$$

accept when the number is divisible by 3.

| Binary #s | decimal | binary |
|---|---|---|
| | 0 | 0 |
| | 1 | 1 |
| | 2 | 10 |
| | 3 | 11 |
| | 4 | 100 |
| | 5 | 101 |
| | 6 | 110 |
| | 7 | 111 |
| | 8 | 1000 |
| | 9 | 1001 |
| | 10 | 1010 |
| | 11 | 1011 |
| | 12 | 1100 |

3 → 6

11 → 110

10110(0)1001(0)

DFA reads from left to right.

How does the value of the string change when we encounter a new letter on the right?

101 → 1010
101 ↪ 1011

When we have x, & put a 0 on the right, the value becomes: 2x

. . . . . x . . . 1 on the right

it becomes 2x+1