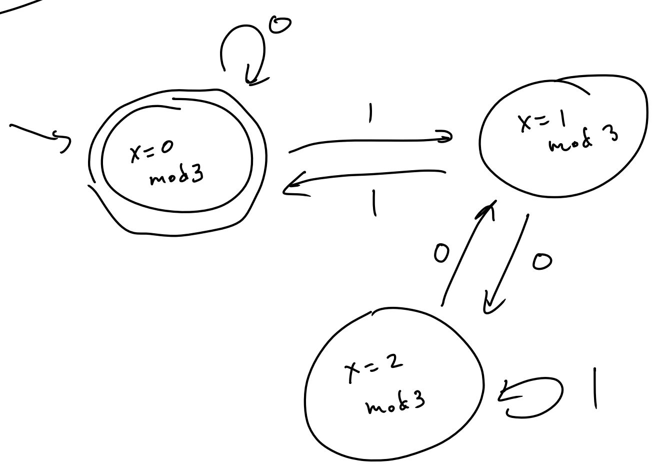
A DFA to accept binary #s that are divisible by 3.

If we've seen x and we read a 0, the new value is $2x$

. . . . 1,

. . . . . . is $2x+1$

0000101



12 is 1100

accepted!

16 is 10000

is rejected

If $x = 0 \bmod 3$, then $x = 3k$, so $2x = 2(3k)$
$= 3(2k) = 0 \bmod 3.$

if we see a 1: it becomes

$2x+1 = 2(3k)+1 = 3(2k)+1 = 1 \bmod 3$

now if $x = 1 \bmod 3$, then $x = 3k+1$

if we see 0, we get $2x = 2(3k+1)$
$= 6k+2$
$= 3(2k)+2 = 2 \bmod 3.$

if we see 1 we get $2x+1 = 2(3k+1)+1$

$$= 6k+2+1$$
$$= 6k+3 = 3(2k+1) = 0 \mod 3$$

if $x = 2 \mod 3$, then $x = 3k+2$

if we see 0, we get $2x = 2(3k+2)$

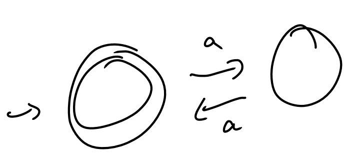
$$= 6k+4$$
$$= 6k+3+1$$
$$= 3(2k+1)+1$$

if we see 1, we get

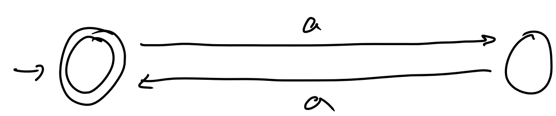
$$2x+1 = 2(3k+2)+1$$
$$= 6k+4+1 = 6k+5 = 2 \mod 3$$

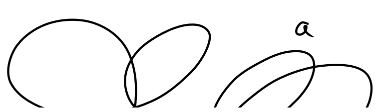
What really <u>is</u> a DFA?

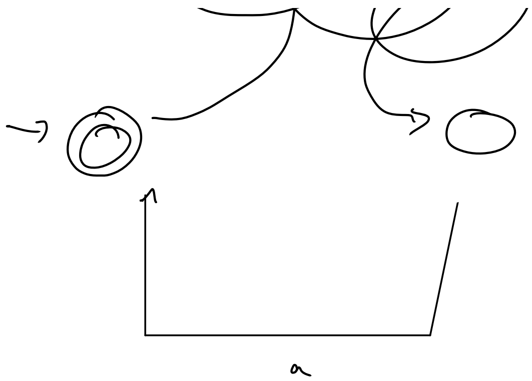
Mathematically we don't like to define things as pictures.



vs







The true nature of a DFA is given by certain sets.

The Formal Definition of a DFA

5 important ingredients:

- Set of all states
- Set of accepting states
- The starting state
- The alphabet
- The transitions

<u>Def</u> A <u>DFA</u> is a 5-tuple

$$M=(Q,\Sigma,\delta,q_0,F)$$

where

Q is a finite set (set of states)

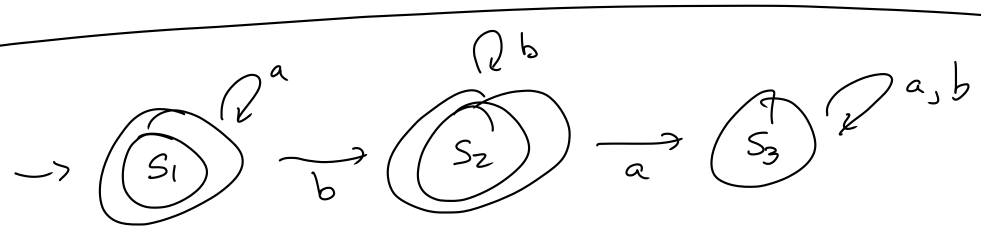
Σ is a finite set (the alphabet)

$\delta : Q \times \Sigma \to Q$ (the transition function)

$\delta(\text{state}, \text{letter}) = \text{state}$

$q_0 \in Q$ (the starting state)

$F \subseteq Q$ (the set of accepting states)



Write this DFA formally as a 5-tuple.

$Q = \{s_1, s_2, s_3\}$

$\Sigma = \{a, b\}$

$q_0 = s_1$

$F = \{s_1, s_2\}$

So the DFA is:

$M = (\{s_1, s_2, s_3\}, \{a, b\}, \delta, s_1, \{s_1, s_2\})$

where δ is:

$\delta(s_1, a) = s_1$

$\delta(s_1, b) = s_2$

$\delta(s_2, a) = s_3$

$\delta(s_2, b) = s_2$

$\delta(s_3, a) = s_3$

$$\delta(s_3, b) = s_3$$

We'll define $L(M)$ formally.

it involves doing δ over & over.

We have the <u>extended transition function</u>

$$\delta^*(s_1, aba) = s_3$$

$$\delta^* : Q \times \Sigma^* \to Q$$

the answer is where we end up after reading the whole string.

Simple properties:

- $\delta^*(q, \varepsilon) = q$
- if $|x| = 1$, then $\delta^*(q, x) = \delta(q, x)$
- $\delta^*(q, xy) = \delta^*(\underbrace{\delta^*(q, x)}_{\text{where we end up after } x}, y)$