A DFA is

$$M = (\underbrace{Q}_{\text{set of states}}, \underbrace{\Sigma}_{\text{alphabet}}, \underbrace{\delta}_{\text{transition function}}, \underbrace{q_0}_{\text{starting state}}, \underbrace{F}_{\text{set of accepting states.}})$$

$$\delta : Q \times \Sigma \to Q$$

$$\delta(s_1, a) = s_3$$

"The extended transition function"

$$\delta^*(s_2, abaa) = s_4$$

$$\delta^*(q, \varepsilon) = q$$

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$$

$x \in \Sigma^*$ is accepted when: if we start in the starting state and read x, we end up in an accepting state.

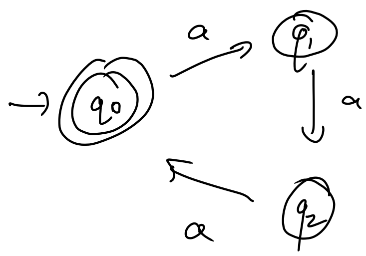
Formally: x is accepted when:

$$\boxed{\underbrace{\delta^*(q_0, x)}_{\text{state we arrive in after reading } x} \underbrace{\in F}_{\text{is an accepting state}}}$$

~~Def~~ If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA,
then the language accepted by M is

$$L(M) = \{ x \in \Sigma^* \mid \delta^*(q_0, x) \in F \}$$

This formal approach is necessary to prove stuff about DFAs.



on $\Sigma = \{a\}$

Let's prove that

$$L(M) = \{ a^n \mid n \equiv 0 \mod 3 \}$$

Want to show $\{ a^n \mid n \equiv 0 \mod 3 \} = \{ a^n \mid \delta^*(q_0, a^n) \in \{q_0\} \}$
$= \{ a^n \mid \delta^*(q_0, a^n) = q_0 \}$

$$\boxed{\text{Want show } n \equiv 0 \mod 3 \text{ iff } \delta^*(q_0, a^n) = q_0}$$

what is $\delta^*(q_0, a^n)$?

$\delta(q_0, a) = q_1$
$\delta(q_1, a) = q_2$
$\delta(q_2, a) = q_0$

note: $\delta(q_i, a) = q_{i+1}$
(subscript mod 3)

$$\delta^*(q_i, a^2) = q_{i+2} \pmod 3$$

$$\vdots$$

$$\delta^*(q_i, a^n) = q_{i+n} \pmod 3$$

for $i=0$, $\delta^*(q_0, a^n) = q_n \pmod 3$

So $\delta^*(q_0, a^n) = q_0$ iff $n = 0 \bmod 3$

Shewn!

Regular Languages

A regular language is one which is accepted by some DFA.

Which languages are regular?

We have many examples of regular languages, can we combine them & get more?

Very simple: Complements

if L is a language, $\overline{L}$ is the complement,

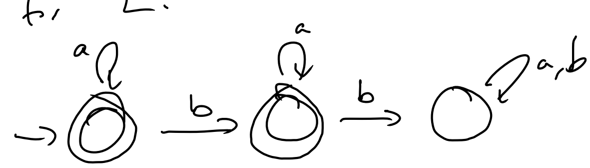
$$\overline{L} = \{ x \in \Sigma^* \mid x \notin L \}$$

if L is regular, what about $\overline{L}$?

ex) $L = \{ x \mid x \text{ uses at most } 1 \ b \}$

$\overline{L} = \{ x \mid x \text{ uses 2 or more b's} \}$

a DFA for L:



to get DFA for $\overline{L}$, we swap the accepting & rejecting states.

if $M = (Q, \Sigma, \delta, q_0, F)$ has language L,

then $(Q, \Sigma, \delta, q_0, \overline{F})$ has language $\overline{L}$.

Thm If L is regular,

then $\overline{L}$ is regular.