Colloquium Tonight! 5pm on Zoom

Closure properties for regular languages

If L is a regular language, then
$\overline{L}$ is also a regular language.

regular langs are "closed under complement"

If we have a DFA for L,
the DFA for $\overline{L}$ is obtained by swapping
all accepting/rejecting states.

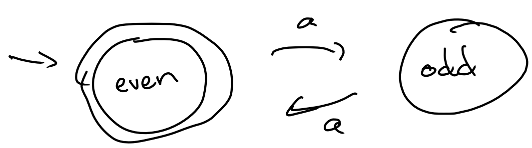
Intersections

Thm If L_1 & L_2 are regular languages, then
$L_1 \cap L_2$ is a regular language.

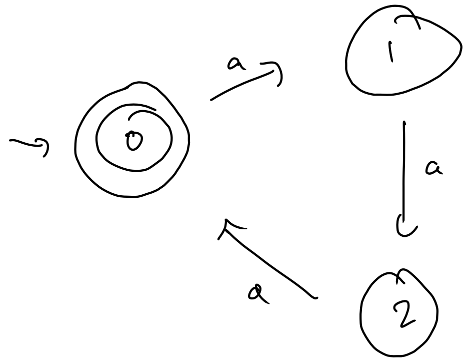
On DFAs, this is done using "the product construction"

Ex $\Sigma = \{a\}$
$L_1 = \{ a^n \mid n \text{ is even} \}$ want $L_1 \cap L_2$
$L_2 = \{ a^n \mid n \text{ is div. by } 3 \}$

M_1 :

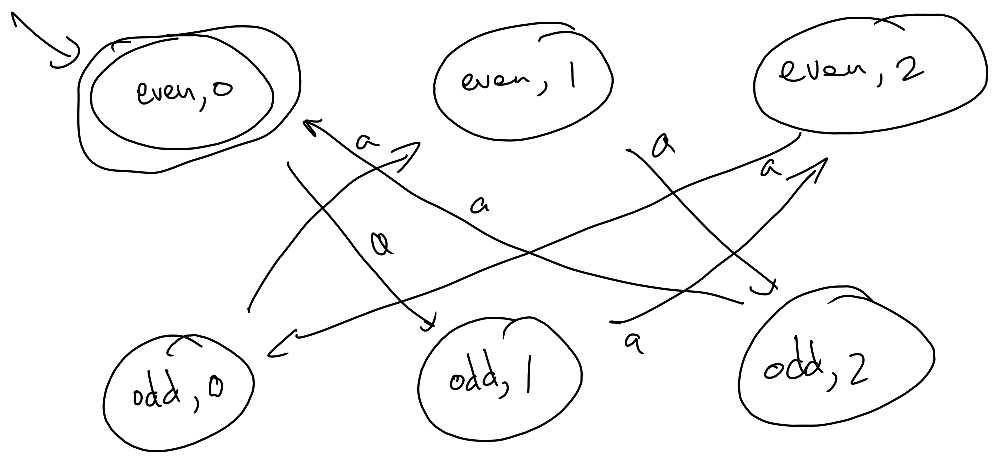


M_2 :



We want to combine these two "with AND"

Combine the states in pairs:

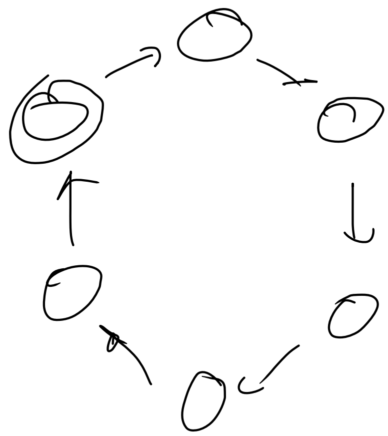


The states are pairs of states from the original machines, transitions apply the letter to each part of pair, starting state is the pair obtained from the starting states of M_1 & M_2

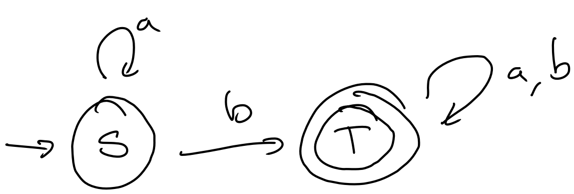
Accepting states are any pair of states

which are both accepting in M_1 & M_2.

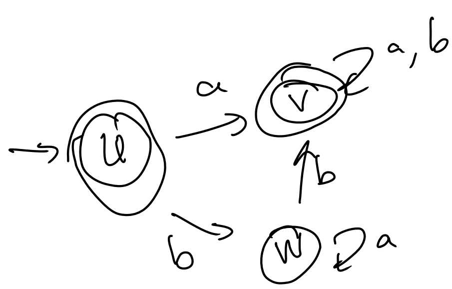
ex) is equivalent to



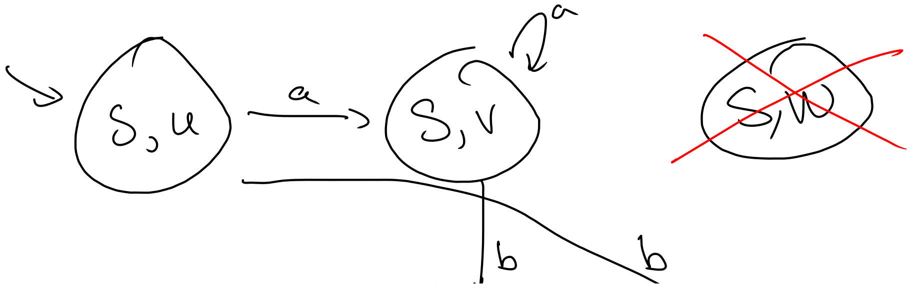
M_1:

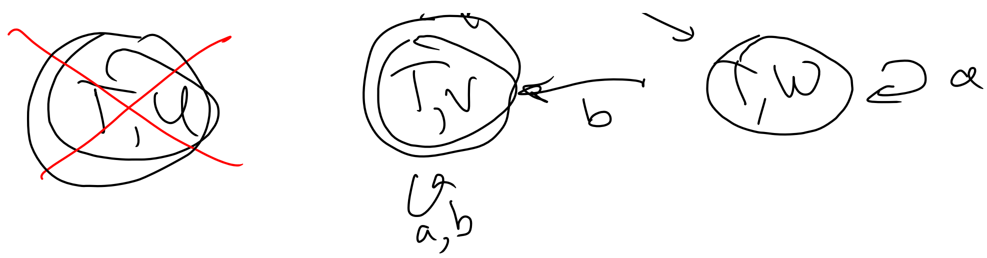


M_2:

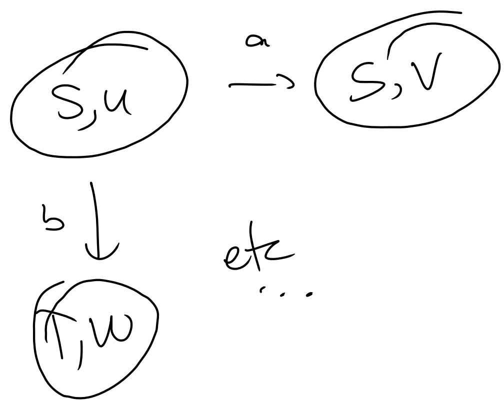


Make a DFA for the intersection.





Some pairs may be unreachable from the starting state.



"Prove the product construction"

Say $M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$

$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$

Then the product construction results in:

$$M_3 = (Q \times R, \Sigma, \delta, (q_0, r_0), F_1 \times F_2)$$

where $\delta((q,r), a) = (\delta_1(q,a), \delta_2(r,a))$

Our proof should show that

$$L(M_3) = L(M_1) \cap L(M_2)$$

[Remember $L(M) = \{ x \in \Sigma^* \mid \delta^*(q_0, x) \in F \}$]

Pf

$$L(M_3) = \{ x \in \Sigma^* \mid \delta^*((q_0, r_0), x) \in F_1 \times F_2 \}$$

$$= \{ x \in \Sigma^* \mid (\delta_1^*(q_0, x), \delta_2^*(r_0, x)) \in F_1 \times F_2 \}$$

$$= \{ x \in \Sigma^* \mid \delta_1^*(q_0, x) \in F_1 \text{ and } \delta_2^*(r_0, x) \in F_2 \}$$

$$= \{ x \in \Sigma^* \mid \delta_1^*(q_0, x) \in F_1 \} \cap \{ x \in \Sigma^* \mid \delta_2^*(r_0, x) \in F_2 \}$$

$$= L(M_1) \cap L(M_2)$$

Shown!