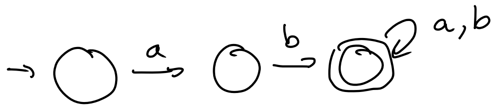
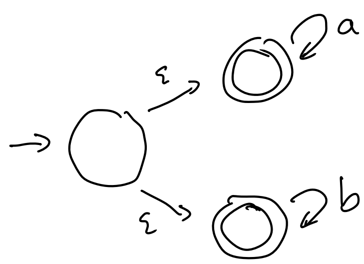


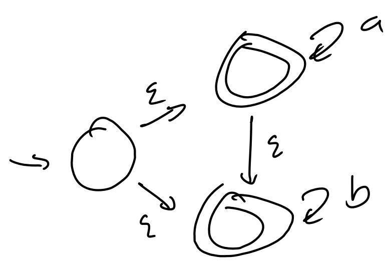
Accepts anything starting with ab.

$\{ abx \mid x \in \{a,b\}^* \}$

NFAs also allow a "spontaneous transition" called "ε-transition"

All strings like a^n or b^n





accepts $a^n b^m$

Formal Description of NFA.

<u>Def</u> An NFA is a 5-tuple:

$$M = (Q, \Sigma, \delta, q_0, F)$$

where:

S

$\mathcal{P}(S)$

Q is a finite set of states

Σ is a finite set (the alphabet)

δ is a function

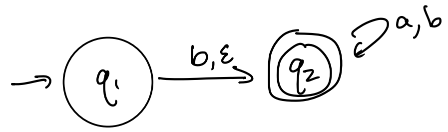
$$\delta : \underbrace{Q \times (\Sigma \cup \{\varepsilon\})}_{\text{for } \delta \text{, we plug in a pair}} \longrightarrow \mathcal{P}(Q)$$

$\delta(\text{state}, \text{letter or } \varepsilon) = \{\text{states}\}$ set of states

$q_0 \in Q$ (the starting state)

$F \subseteq Q$ (set of accepting states)

Ex/



Formally, it's

$$M = (\{q_1, q_2\}, \{a, b\}, \delta, q_1, \{q_2\})$$

where δ is:

$\delta(q_1, a) = \emptyset$

$\delta(q_1, b) = \{q_2\}$

$\delta(q_1, \varepsilon) = \{q_2\}$

$\delta(q_2, a) = \{q_2\}$

$\delta(q_2, b) = \{q_2\}$

$\delta(q_2, \varepsilon) = \emptyset$

[in DFA, x is accepted means $\delta^*(q_0, x) \in F$]

In a NFA, $\delta(q_0, a)$ is a set.

for a string x, $\delta^*(q_0, x)$ is the set of all possible destinations if we start in q_0 and do x.

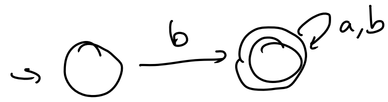
x is accepted if any of these destinations is an accepting state.

if any state in $\delta^*(q_0, x)$ is in F.

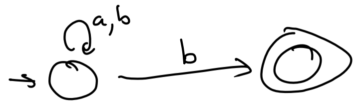
$$L(M) = \{ x \in \Sigma^* \mid \delta^*(q_0, x) \text{ contains an element of } F \}$$

$$\boxed{L(M) = \{ x \in \Sigma^* \mid \delta^*(q_0, x) \cap F \neq \emptyset \}}$$

Unions with NFAs:

M_1: 

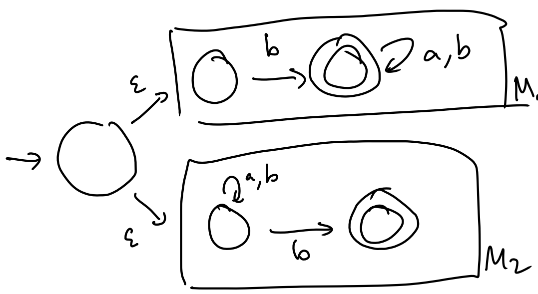
$L(M_1) = \{bx \mid x \in \{a,b\}^*\}$

M_2: 

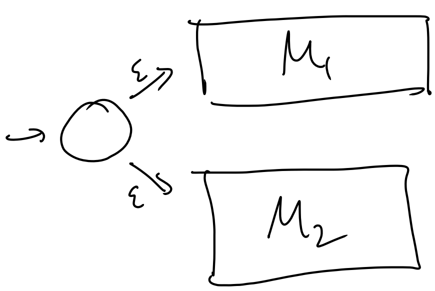
$L(M_2) = \{xb \mid x \in \{a,b\}^*\}$

An NFA for $L(M_1) \cup L(M_2)$:

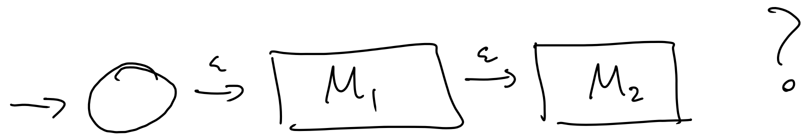
"starts with b
or ends with b"



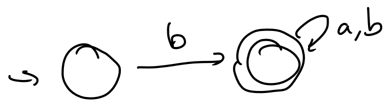
Generally if we have M_1 & M_2,
to get $L(M_1) \cup L(M_2)$, do:

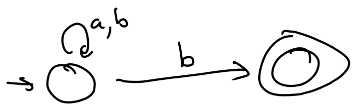


What do we get from



This accepts strings like xy,
where $x \in L(M_1)$ & $y \in L(M_2)$

M_1: 

M_2: 

This gives the concatenation of the languages.

$$L(M_1)L(M_2) = \{xy \mid x \in L(M_1) \text{ and } y \in L(M_2)\}$$

To draw the NFA for a complicated language, maybe think of it as a union.

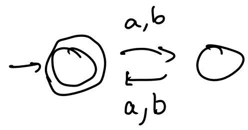
Ex| Make a NFA which accepts any string with even length or an even # of b's.

Think of it like a union:

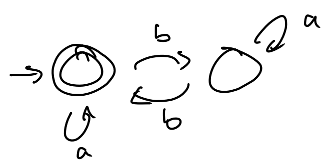
$\{\text{even length}\} \cup \{\text{even \# } b\text{'s}\}$

$L(M_1)$ $\qquad\qquad$ $L(M_2)$

M_1:



M_2:



Final answer: