Brzozowski Derivative

For some language L,

$$\frac{d}{da} L = \{ x \mid ax \in L \}$$

$$\frac{d}{da} \{ ab^n \} = \{ b^n \}$$

$$\frac{d}{db} \{ ab^n \} = \emptyset$$

$$\frac{d}{da} \{ ab^n ab^n \} = \{ b^n ab^n \}$$

$$\frac{d}{da} \{ a^n \} = \{ a^{n-1} \}$$ ← OK, but strange

$$\cancel{= \{ a^n \mid n \geq 1 \}}$$

$$\frac{d}{da} \{ a^n \} = \{ a^n \}$$

$$\frac{d}{da} \{ (ab)^n \} = \{ b(ab)^n \}$$

$$= \{ (ba)^n b \}$$

$\cancel{a}babab$

For any string x,

$$\frac{d}{dx} L = \{ s \mid xs \in L \}$$

$$\frac{d}{d(ab)} \{(ab)^n\} = \{(ab)^n\}$$

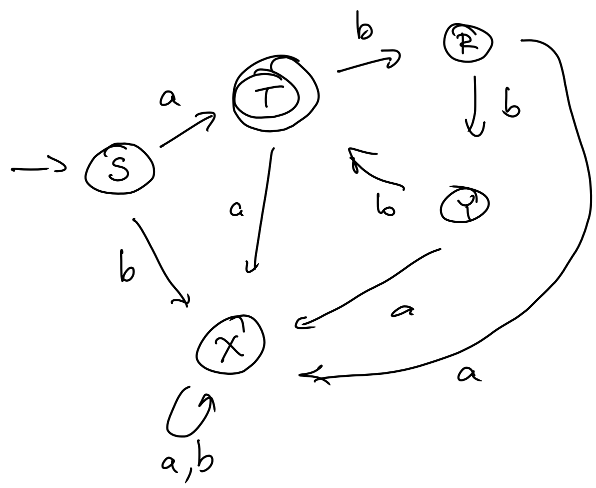
$$\frac{d}{d(aba)} \{(ab)^n\} = \{b(ab)^n\}$$

$$\frac{d}{da^2} \{(ab)^n\} = \emptyset$$

E = english words

$$\frac{d}{d(bop)} E = \{\varepsilon, per, pers, ped, ping, s\}$$

M:



$L(M) = \{a(b^3)^n\}$

If I move the start state to T,

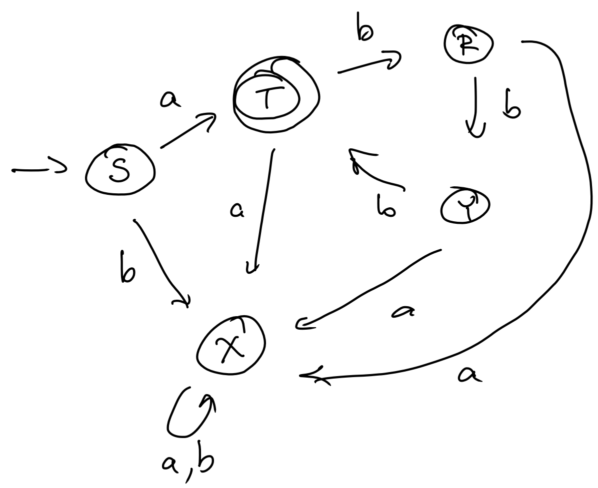
then L becomes $\{(b^3)^n\} = \frac{d}{da} L(M)$

Moving the start state thru an edge labeled a changes the language into $\frac{d}{da} L(M)$.

If we move start state to X, then the lang should become

$$\frac{d}{db} L(M) = \frac{d}{db} \{a(b^3)^n\} = \emptyset$$

$\frac{d}{dx} L(M)$ corresponds to moving the start state through several edges spelling out the string x.



I can tell immediately that

$$\frac{d}{da} L(M) = \frac{d}{d(ab^3)} L(M)$$

For this language, there are only 5 possible derivatives $\frac{d}{dx}$ for various x.

$\frac{d}{dab}\{a(b^3)^n\} = \{b^2(b^3)^n\}$

If some language L has a DFA, there must be only finitely many derivatives.

$P \Rightarrow Q$

Thm If L is regular, then it has only finitely many derivatives

Contrapositive:

Thm If L has infinitely many derivatives, then L is not regular.

Ex $L = \{a^n b^n\}$ is not regular.

Pf We'll show L has ∞-ly many derivatives.

Our derivs will be

$\frac{d}{da}, \frac{d}{da^2}, \frac{d}{da^3}, \ldots$

Let $D_i = \frac{d}{da^i} L$,

then $D_i = \{ a^{n-i} b^n \}$

this has: # of a's is i less than # b's.

These D_i are all different, so L has ∞-ly many different derivatives.

So L is not regular.

Ex| $L = \{ a^n b^n c^n \}$ is not regular.

Pf Let $D_i = \frac{d}{da^i} L = \{ a^{n-i} b^n c^n \}$

here, # of a's is i less than # of b's & c's.

So the D_i are all different, so L is not regular.

Ex| $L = \{ (ba)^n (ab)^n \}$

Let $D_i = \frac{d}{d(ba)^i} L = \{ (ba)^{n-i} (ab)^n \}$

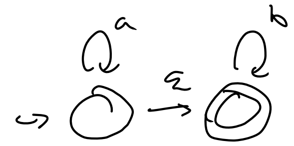
here, # of (ba)'s is i less than # of (ab)'s.

So the D_i are all different,

so L is nonregular.

What happens if L was regular?

$L = \{a^n b^m\}$



(L is regular)

Let $D_i = \frac{d}{da^i} L = \{a^{n-i} b^m\}$

$= \{a^n b^m\}$

These are all the same, so

we cannot conclude L is nonregular.

$L = \{a^n\}$

$D_i = \frac{d}{da^i} L = \{a^{n-i}\} = \{a^n\}$