More nonregular languages!

Thm If L has infinitely many derivatives, then L is nonregular.

Ex $L = \{ab^n a^n\}$

PF Let $D_i = \frac{d}{d(ab^i)} L = \{b^{n-i} a^n\}$

these D_i are all different, since in D_i we have i fewer b's than a's.

So there are ∞-ly many derivatives, so L is nonregular.

The real issue comes from the re-use of the n in $a^n b^n$

vs $a^n b^m$ is regular

$L = \{a^n b^{2n}\}$ is not regular

PF Let $D_i = \frac{d}{da^i} L = \{a^{n-i} b^{2n}\}$

in D_i, # a's is i less than half # b's.

these D_i are all different, so L is non-regular.

Ex $L = \{ a^m b^n a^n \}$

use $D_i = \frac{\partial}{\partial (a^m b^i)} L = \{ b^{n-i} a^n \}$

a^m is bogus here, since m is a variable.

$L = \{ a^m b^n a^n \}$

$D_i = \frac{\partial}{\partial a^i} L = \{ a^{m-i} b^n a^n \} = \{ a^m b^{n-n} \}$

Let $D_i = \frac{\partial}{\partial b^i} L = \{ b^{n-i} a^n \}$

[in L, most strings start with a, but for those where $m=0$, we take $\frac{\partial}{\partial b^i}$ and get $b^{n-i} a^n$]

Then D_i has i fewer b's than a's, so D_i are all different.

$\{ a^n b^n \}$

Ex1 $\{x \mid \#a\text{'s in } x = \#b\text{'s in } x\}$

$a^i(\sim)$

Let $D_i = \frac{d}{da^i} L = \{x \mid \#a\text{'s in } x = (\#b\text{'s in } x) - i\}$

So in D_i, #a's is i fewer than #b's.

So D_i are all different!

Ex1 $\{x \mid \#a\text{'s in } x < \#b\text{'s in } x\}$

Let $D_i = \frac{d}{da^i} L = \{x \mid \#a\text{'s in } x < \#b\text{'s in } x - i\}$

So D_i are all different.

$L = \{x^2 \mid x \in \{a,b\}^*\}$

all squares! aab aab

Let $D_i = \frac{d}{da^i} L = \{y a^i y \mid y \in \{a,b\}^*\}$

[when x begins with a^i, then it's like $x = a^i y$
so $x^2 = a^i y a^i y$]

So D_i is strings with i a's in the middle

these are all different!

$L = \{ x x^R \mid x \in \{a,b\}^* \}$

x^R means reverse of x

$(abaab)^R = baaba$

$D_i = \frac{d}{da^i} L = \{ y y^R a^i \}$

[if x starts with a^i, then $x = a^i y$,

$x x^R = a^i y (a^i y)^R$

$= a^i y y^R a^i$]

D_i ends with a^i, so these are all different.

$\{ b^n a b^n \}$

$\{ a^n b^m \mid 2n < m \}$

$\{ x \bar{x} \mid x \in \{0,1\}^* \}$, bar means "NOT"

$L = \{ b^n a b^n \}$

Let $D_i = \frac{d}{db^i} L = \{ b^{n-i} a b^n \}$

has i fewer b's at the beginning than b's at

the end, so D_i are all different.

$$\{a^n b^m \mid 2n < m\}$$

let $D_i = \frac{\partial}{\partial a^i} L = \{a^{n-i} b^m \mid 2n < m\}$ $\quad \ell = n - i$

i fewer a's is < half the b's.

$$\{a^\ell b^m \mid 2(\ell + i) < m\}$$

a's & # b's satisfy $2(\ell + i) < m$.