Grammars $\longleftrightarrow$ NFA, DFA, regex

$$G: \quad \begin{aligned} S &\to aS \mid T \\ T &\to bT \mid S \mid \varepsilon \end{aligned}$$

$L(G) = \{a,b\}^*$ we can derive any string.

abba

$$S \Rightarrow aS \Rightarrow aT \Rightarrow abT \Rightarrow abbT \Rightarrow abbS$$
$$\Rightarrow abbaS \Rightarrow abbaT \Rightarrow abba$$

Some grammars make nonregular langs

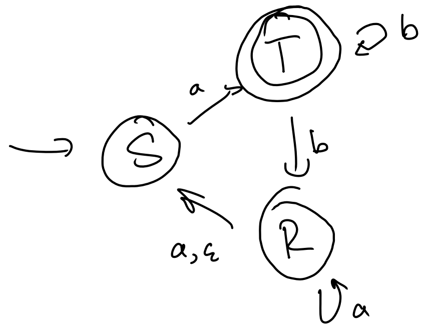
$S \to aSb \mid \varepsilon$ generates $\{a^n b^n\}$

$S \to aSb \mid a$ $\{a^{n+1} b^n\}$

G: $S \to aS \mid aSb$ $L(G) = \emptyset$

So there can be no procedure to convert any grammar to NFA/DFA/regex

It is possible to convert NFA $\to$ grammar



Each state becomes a nonterminal,

Start state becomes S

$S \to aT$

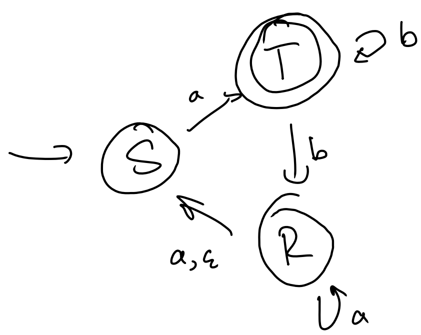
$T \to bT \mid bR \mid \varepsilon$ $\leftarrow$ T is accepting state

$R \to aR \mid aS \mid S$

Each NFA transition $X \xrightarrow{a} Y$

becomes $X \to aY$

if X is accepting, also include $X \to \varepsilon$



$S \to aT$

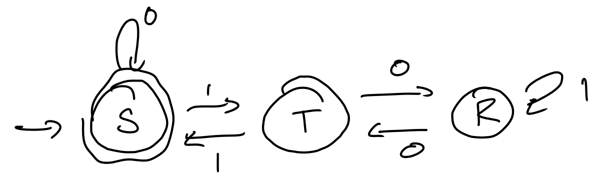
$T \to bT \mid bR \mid \varepsilon$

$R \to aR \mid aS \mid S$

NFA accepts abbaa

can we derive it?

$$S \Rightarrow aT \Rightarrow abT \Rightarrow abbR \Rightarrow abbaS \Rightarrow abbaaT$$
$$\Rightarrow abbaa$$



$$S \to 1T \mid 0S \mid \varepsilon$$
$$T \to 0R \mid 1S$$
$$R \to 0T \mid 1R$$

<u>Thm</u> Every regular language can be generated by a grammar.

These regular languages can all be generated by "<u>single step</u>" grammars.

<u>Def</u> A grammar is single-step when all productions look like:

$X \to aY$ where X & Y are nonterminal (perhaps $X=Y$)

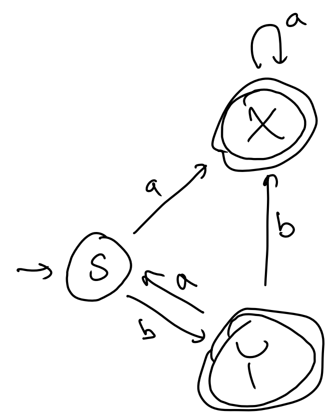
or $X \to \varepsilon$ a is terminal.

A single-step grammar can be converted to NFA

$S \to aX \mid bY$

$X \to aX \mid \varepsilon$

$Y \to aS \mid bX \mid \varepsilon$



A bit fancier:

<u>Def</u> A <u>right-linear grammar</u> is one where all productions look like

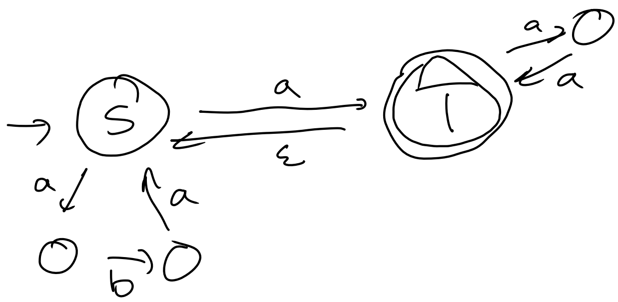
$X \to zY$ where z is a terminal string.

or $X \to \varepsilon$.

$S \to abaS \mid aT$

$T \to aaT \mid S \mid \varepsilon$

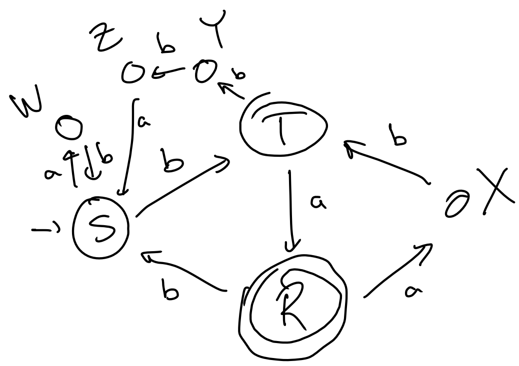
These also can be converted into NFA:



$S \to abS \mid bT$

$T \to bbaS \mid aR$

$R \to bS \mid abT \mid \varepsilon$



$S \to aW \mid bT$

$T \to bY \mid aR$

$R \to aX \mid bS \mid \varepsilon$

$X \to bT$

$Y \to bZ$

$Z \to aS$

$W \to bS$

this is single step!

Actually, any right-linear can be converted into a single-step grammar.

Context-Free grammars (CFG)

Def A CFG is one in which the left side of every production is a single nonterminal.

like $X \to abXY$

NOT $aX \to bY$

In a CFG, the productions are like substitution rules.

$X \to abX$ means X can become abX.

$a\boxed{X} \to a\boxed{bY}$ means X can become bY, but only in a particular context