CFL

context free languages

$S \to aS \mid bT$
$T \to aT \mid \varepsilon$

is a context-free grammar.

Lang. is $a^n b a^m$

$S \to aS \mid bT$
$abT \to abS$
$T \to \varepsilon$

not a context-free grammar.

Remember any regular lang. has a grammar; these grammars are CFG

↑ context-free grammar

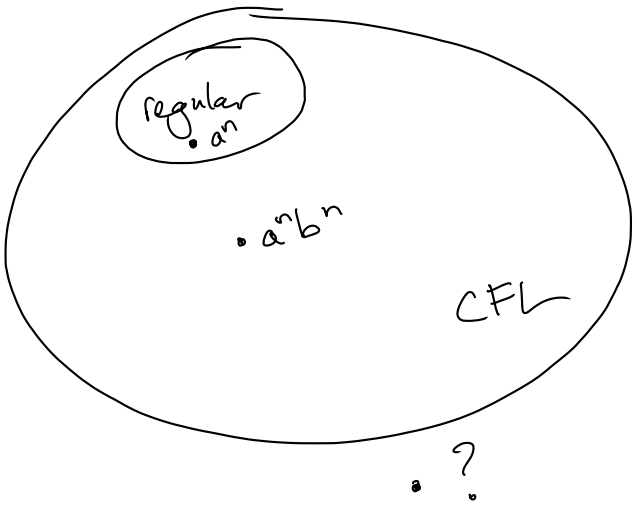
So:

Thm Any regular lang is a CFL.

But some CFL are not regular: $a^n b^n$

$S \to aSb \mid \varepsilon$

There is a hierarchy of languages — Noam



"The Chomsky hierarchy"

Making a CFG for some language

- Regular langs - make NFA, then convert.
- balancing like $a^n b^n$ or $a^n b a^n b$

each a is "balanced" with b.
create them at the same time, from the middle.

$$S \to aSb \mid \varepsilon$$

(not $S \to abS \mid \varepsilon$) ← generates $(ab)^n$

Try $\{a^n b^{2n}\}$ each a balances with bb

$$S \to aSbb \mid \varepsilon$$

$\{a^n b^{n+1}\}$ each a balances with b, but 1 extra b.

$$S \to aSb \mid b$$

$\{a^n b^m \mid m > n\}$ each a balances with b, but 1 or more extra b's.

$$S \to aSb \mid bT$$
$$T \to bT \mid \varepsilon$$

$$S \to aSb \mid T$$
$$T \to bT \mid b$$

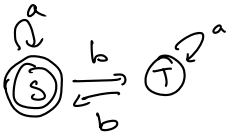
$a^n b a^n$

$$S \to aSa \mid b$$

Concatenations

$\{a^n b^n x \mid x$ has an even # of b's$\}$

regular



$S \to aSb \mid \varepsilon$

$S \to bT \mid aS \mid \varepsilon$
$T \to aT \mid bS$

To concatenate:

$$S \to S_1 S_2$$

$$\begin{cases} S_1 \to aS_1b \mid \varepsilon \\ S_2 \to bT \mid aS_2 \mid \varepsilon \\ T \to aT \mid bS_2 \end{cases}$$

$a^n b a^n b$

$S \to aSa \mid b$ ↑ ↖ $S \to b$

$$\left.\begin{array}{l} S \to S_1 S_2 \\ S_1 \to aS_1a \mid b \\ S_2 \to b \end{array}\right\} \quad \begin{array}{l} S \to S_1 b \\ S_1 \to aS_1a \mid b \end{array}$$

Unions do like

$$S \to S_1 \mid S_2$$

$$S_1 \to \sim$$

$$S_2 \to \sim$$

$L = \{a^n b^n\} \cup \{x \mid x \text{ has an even \# b's}\}$

$$S \to S_1 \mid S_2$$

$$S_1 \to aS_1b \mid \varepsilon$$

$S_2 \to bT \mid aS_2 \mid \varepsilon$

$T \to aT \mid bS_2$

$\{a^n b^m \mid m > n\} = \{a^n b^n b^k \mid k > 0\}$

$S \to S_1 S_2$

$S_1 \to aS_1 b \mid \varepsilon$

$S_2 \to bS_2 \mid b$

$\{a^n b^m \mid n \neq m\}$

$= \underbrace{\{a^n b^m \mid n < m\}}_{S_1} \cup \underbrace{\{a^n b^m \mid m < n\}}_{S_2}$

$S \to S_1 \mid S_2$

$S_1 \to aS_1 b \mid T$

$T \to bT \mid b$

$S_2 \to aS_2 b \mid R$

$R \to aR \mid a$