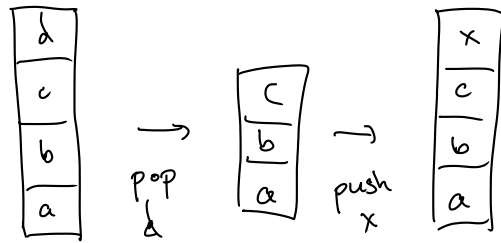


Stack Machines



$dcd a \xrightarrow{\text{pop } d} cda \xrightarrow{\text{push } x} xcda$

"top" is the left end.

<u>Ex</u>	<u>read</u>	<u>pop</u>	<u>push</u>	
1.	a	S	SX	← if I read a, I can pop S & push SX
2.	b	X	ϵ	
3.	ϵ	S	ϵ	← without reading a letter, I can pop S & push nothing

Our stack begins with S,

a string is accepted if we can read the whole string and end with a empty stack

Is aabb accepted?

	<u>read</u>	<u>pop</u>	<u>push</u>
1.	a	S	SX
2.	b	X	ϵ
3.	ϵ	S	ϵ

← The language is $\{a^n b^n\}$

			abab		
<u>input</u>	<u>stack</u>		<u>input</u>	<u>stack</u>	
aabb	S	start	abab	S	
a abb	<u>SX</u>	#1	a bab	SX	#1
a a bb	<u>SXX</u>	#1	a bab	X	#3
a a bb	XX	#3	a bab	ϵ	#2
a a b b	X	#2	stuck! abab is <u>rejected</u>		
a a b b	ϵ	#2			

Write them like:

$$(aabb, S) \mapsto_1 (abb, SX) \mapsto_1 (bb, SXX) \mapsto_3 (bb, XX) \\ \mapsto_2 (b, X) \mapsto_2 (\epsilon, \epsilon)$$

each pair is an "instantaneous description" (ID)

(
 ↑
 piece of
 the input remaining
 to be read.
 ↑
 stack

X is accepted when we can do:

$$(x, S) \mapsto \dots \mapsto (\varepsilon, \varepsilon)$$

	read	pop	push
1.	a	S	X
2.	b	X	Y
3.	b	Y	S
4.	ε	S	ε

$$(abbabb, S) \mapsto \dots \mapsto (abb, S)$$

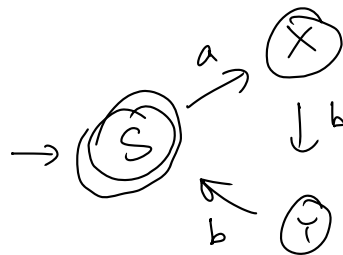
try abb:

$$(abb, S) \mapsto_1 (bb, X) \mapsto_2 (b, Y) \mapsto_3 (\varepsilon, S) \\ \mapsto_4 (\varepsilon, \varepsilon) \text{ accepted!}$$

try aab

$$(aab, S) \mapsto_1 (ab, X) \quad \times \\ (aab, S) \mapsto_4 (aab, \varepsilon) \quad \times \quad \text{rejected.}$$

This stack mach. can be converted to NFA.



lang. is $(abb)^*$
 $\{(abb)^n\}$

$$L = \{ x \in \{a,b\}^* \mid \# \text{ of } a\text{'s} = \# \text{ of } b\text{'s} \}$$

idea: my stack will always look like:

XXXXS or XS
or YYS

X represents one extra a,

Y - - - - - b

A typical derivation would be like:

$$(abbbabaa, S) \mapsto (bbbabaa, XS)$$

$$\mapsto (bbabaa, S) \mapsto (babaa, YS)$$

$$\mapsto (abaa, YYS) \mapsto (baa, YS)$$

$$\mapsto (aa, YYS) \mapsto (a, YS)$$

$$\mapsto (\epsilon, S) \mapsto (\epsilon, \epsilon)$$

	read	pop	push
		S	XS
1	a	X	XX
2	a	Y	ϵ
3	a	S	YS
4	b	X	ϵ
5	b	Y	YY
6	b	S	ϵ
7	ϵ		

give a derivation
for
abbaba

$$\begin{aligned}
 (abbaba, S) &\mapsto_1 (bbaba, XS) \mapsto (baba, S) \\
 &\mapsto (aba, YS) \mapsto (ba, S) \mapsto (a, YS) \\
 &\mapsto (\epsilon, S) \mapsto (\epsilon, \epsilon) \quad \checkmark
 \end{aligned}$$