Turing Machines

Can we prove everything based on axioms?

Is there one master procedure which could be used to prove anything?

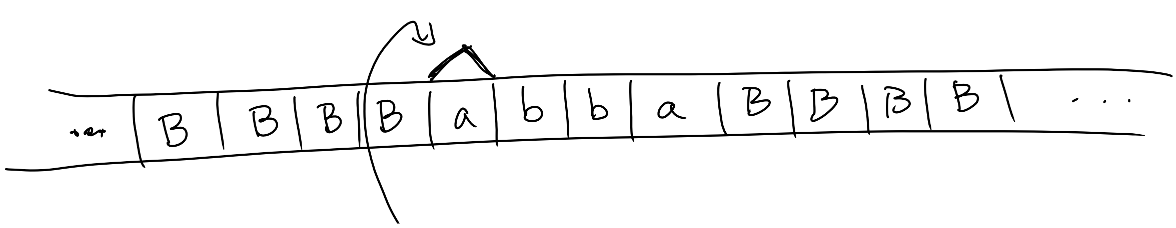
Alonzo Church: A procedure is anything which can be expressed as a recursive function of λ-terms.

Alan Turing: A procedure is anything which can be mechanized.

Turing described the Universal Machine

we call it a Turing Machine (TM)

The TM has states, plus the tape



B is "blank"

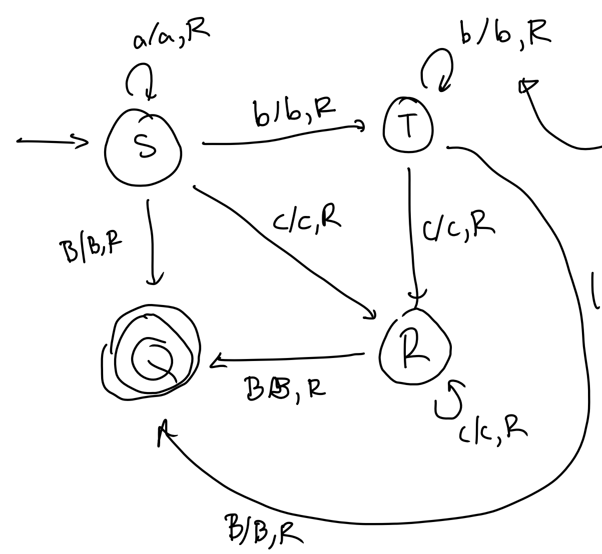
the "head"

We begin with all blank, except the input string, with the head pointing at the 1st letter of the input.

The TM in each step will read the tape where the head is, changes that symbol, and moves one spot Left or Right, and changes state.

If the TM ever enters an accepting state, we stop and the string is accepted.

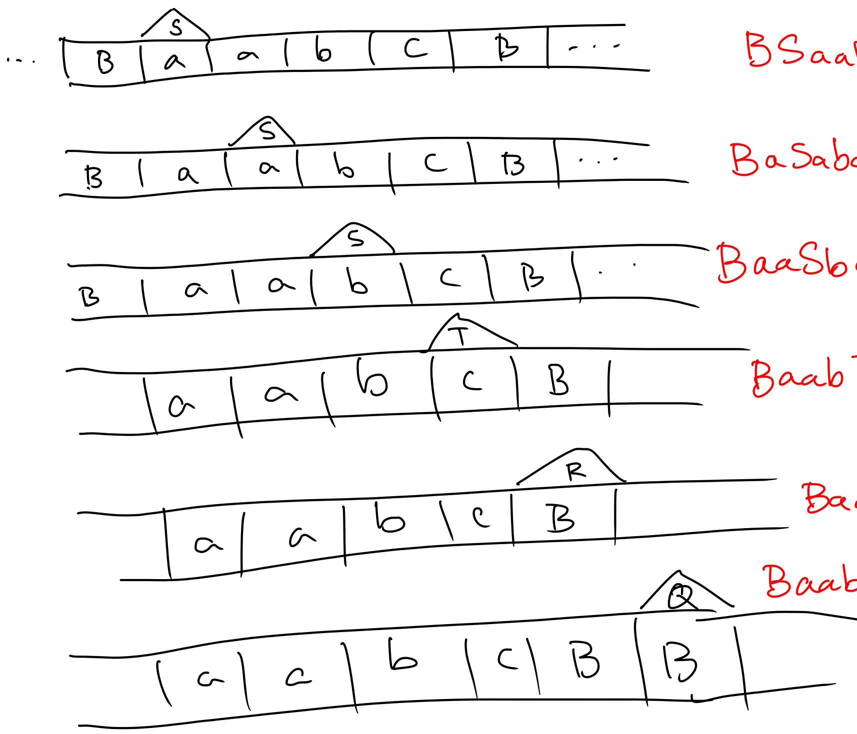
Ex1



read b, replace it with b, then move R.

label like x/y, L means read x, write y, move L

Is aabc accepted?



BSaabcB

BaSabcB

BaaSbcB   ID's

BaabTcB

BaabcRB

BaabcBQB

accept!

if we get stuck, the string is rejected.

The language of this TM is

$a^*b^*c^*$

$\{a^n b^m c^k\}$

This computation was

$BSaabcB \vdash BaSabcB \vdash BaaSbcB$

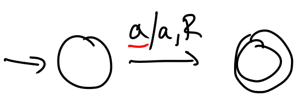
$\vdash BaabTcB \vdash BaabcRB$

$\vdash BaabcBQB$

Simple examples

A TM for $\{ax \mid x \in \{a,b\}^*\}$.

"anything that starts with a"

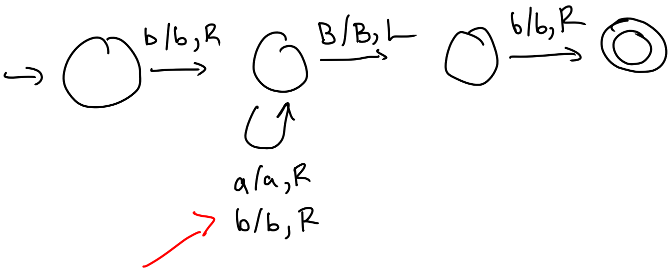


accepts if we start with a,
jams if we start with b
or if it's ε.

~~$\{bxb \mid x \in \{a,b\}^*\}$~~

starts & ends with b.

if it starts with b, move R all the way until we hit B,
then back L once, then if it's b, accept.



continue moving R
thru any a's & b's

$\{abxcya \mid x,y \in \{a,b\}^*\}$