Math 1015: Homework #4

Question 1. Consider this weighted voting system: $[18 : 6, 4, 4, 3, 2, 1]$. In each part, briefly explain (don't just say "yes" or "no"):

a) Are there any dictators?

b) Are there any voters with veto power?

c) Are there any dummies?

Question 2. In each part, invent a different example of a weighted system (like [?:?????]) having:

a) a dictator

b) someone with veto power who is not a dictator

c) more than one voter with veto power

d) a voter with veto power, and at least one dummy

Question 3. Look up the numbers for the weights used in the US Electoral College system. (Try Wikipedia "United States Electoral College", scroll down to section "Current electoral vote distribution".)

In each part, briefly explain (don't just say "yes" or "no"):

a) What is the quota in this system?

b) Is any state a dictator in this system?

c) Does any state have veto power?

d) Is any state a dummy?

Question 4. Consider this weighted system:

$$[20 : x, 8, 8, 4, 2]$$

(here, the x is unknown— it might be less than 8) As usual, give a few words to explain your answer.

a) What is the smallest value of x that makes it a dictator? (Or perhaps say that x cannot be a dictator no matter what.)

b) What is the smallest value of x that makes it have veto power? (Or perhaps say that x cannot have veto power no matter what.)

c) What is the biggest value of x that makes it a dummy? (Or perhaps say that x cannot be a dummy no matter what.)

Question 5. Let's say we have 4 people voting, and their weights are 8, 5, 4, and 2. What are the allowable values for the quota, according to the inequalities we discussed in class?

Question 6. Compute the Shapley-Shubik power index for $[15 : 10, 7, 3]$.

Question 7. Compute the Shapley-Shubik power index for [12: 8, 8, 4].