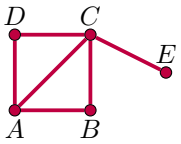
Math 1015: Homework #9

Question 1. Consider this graph:



a) Does this graph have an Euler circuit? Either demonstrate an Euler circuit (draw it), or say why none exists.

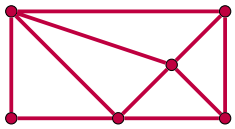
b) Does this graph have an Euler path? Either demonstrate an Euler path, or say why none exists.

c) Use this graph as a specific example to demonstrate Euler's sum-of-degrees theorem.

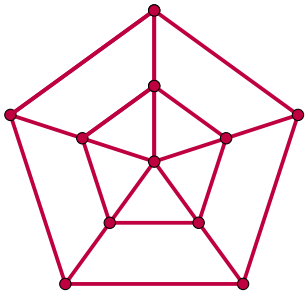
Question 2. Imagine I have 5 friends, and some of them are friends with each other too. My friends are Tony, Erin, Chuck, Patty, and Shawn. (Yes these are real people.) Tony is friends with 3 of these 5 people, Erin is friends with 2 of them, Chuck is friends with 1 of them, and Patty is friends with 3 of them. Shawn is not friends with any of them. Explain using Euler's sum-of-degrees theorem why this is impossible.

Question 3. For each of these graphs, say whether or not it has an Euler circuit, and whether or not it has an Euler path. If it has either one, draw it in. (Make sure the path you draw is legible.)

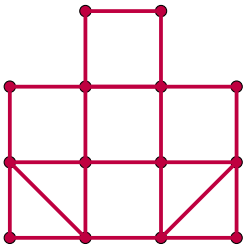
a)



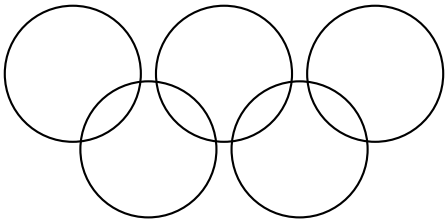
b)



c)

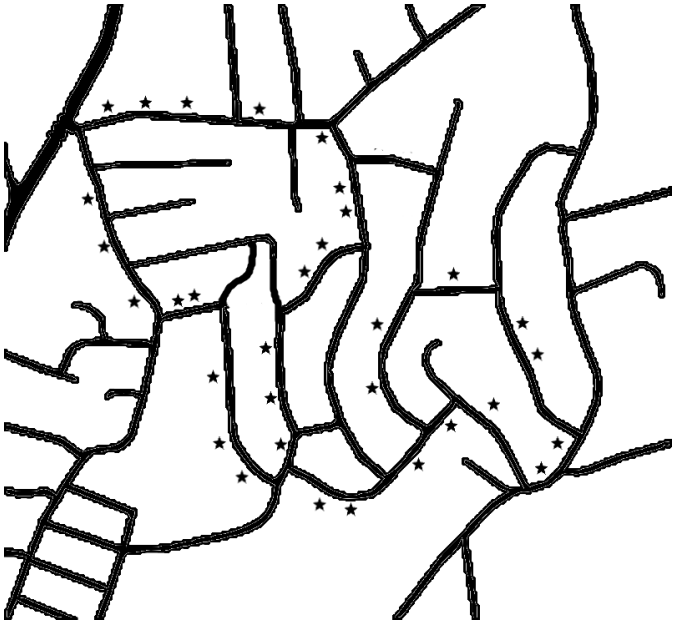


Question 4. Let's say I want to paint this pattern on the floor in my dorm:



Is it possible to paint this all at once in a single stroke without re-tracing over any segments? If it is possible, demonstrate how I would do it. If it is not possible, explain why not.

Question 5. My daughter needs to deliver her Girl Scout Cookies in this neighborhood:



The lines are streets, and each star is a house where she needs to bring her cookies.

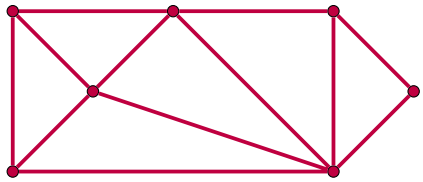
a) Translate this picture into a graph, including only the streets that my daughter has deliveries on. (Most streets will not be included on your graph.)

b) Does this graph has an Euler circuit? Does it have an Euler path?

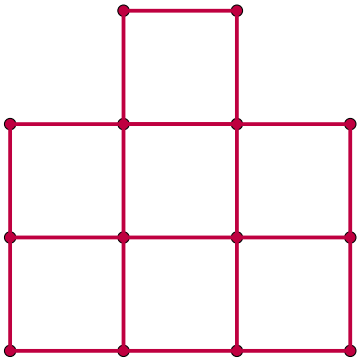
c) Give some real-world interpretation of your answer in part b in terms my 10 year old daughter would understand. If you like, begin your answer with "OK, honey,"

Question 6. For each part find a minimum duplication circuit, and a minimum duplication path:

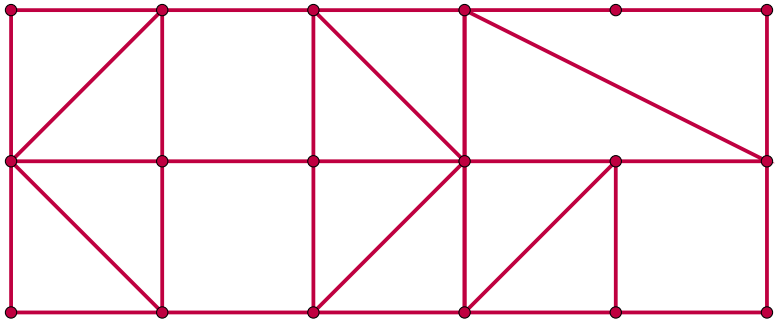
a)



b)



Question 7. Both parts are about this graph:



a) Please explain why there is an Euler path.

b) Please show which edges you would duplicate in order to form a minimum duplication circuit.

c) Please explain why it's not very interesting to find a minimum duplication path, given what you've already done.