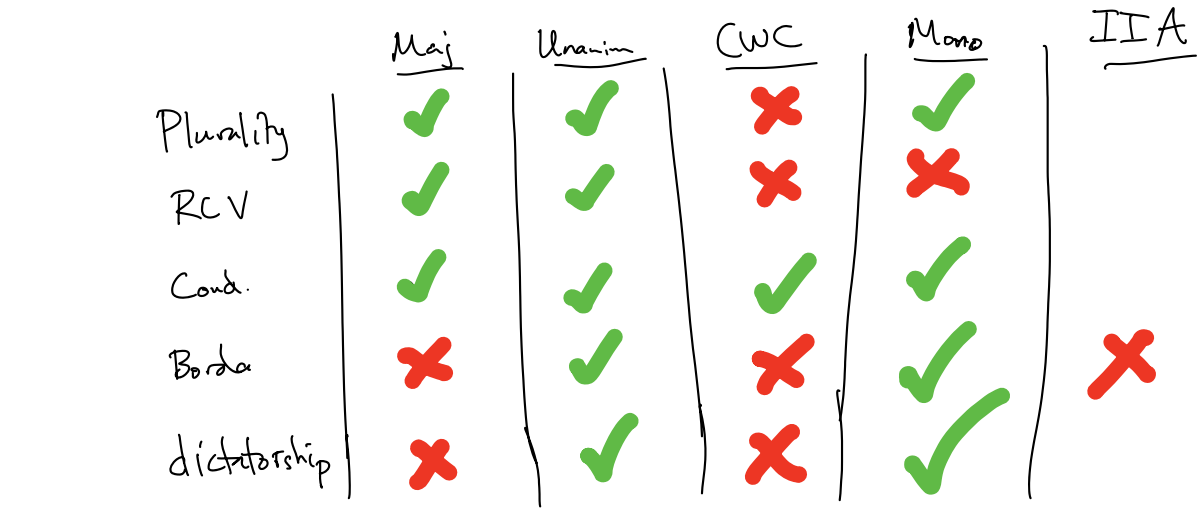


A new criterion: Monotonicity criterion

"monotone" means "always the same direction"

About people changing their minds, like if I was gunna put A on top, but then I swap it with B.

(This should help B).

Say we have:

| 5 | 4 | 3 | 1 | with Borda. |
|---|---|---|---|---|
| A | C | B | B | 2 |
| B | B | C | A | 1 |
| C | A | A | C | 0 |

A: $5 \cdot 2 + 4 \cdot 0 + 3 \cdot 0 + 1 \cdot 1$

$10 + 0 + 0 + 1 = 11$

B: $5 \cdot 1 + 4 \cdot 1 + 3 \cdot 2 + 1 \cdot 2$

$5 + 4 + 6 + 2 = 17$ B wins!

C: $5 \cdot 0 + 4 \cdot 2 + 3 \cdot 1 + 1 \cdot 0$

$0 + 8 + 3 + 0 = 11$

If the C, B, A's change to B, C, A )

then B gets even more pts, so B still wins

this change is "boosting the winner"

Using RCV:

| 5 | 4 | 3 | 1 |
|---|---|---|---|
| A | C | B | B |
| B | B | C | A |
| C | A | A | C |

Rd 1 A: 5

~~B: 4~~ elim B&C

C: 4

A wins with RCV.

Imagine B, (A), C changes to (A), B, C

Now RCV:

| 5 | 4 | 3 | 1 |
|---|---|---|---|
| A | C | ~~B~~ | A |
| ~~B~~ | ~~B~~ | C | ~~B~~ |
| C | A | A | C |

Rd 1

A: 6

~~B: 3~~

C: 4

elim. B.

Rd 2 A: 6

C: 7

C wins!

Sometimes in RCV, boosting up the winner will cause them to lose.

---

## Monotonicity Criterion

Def A system satisfies monotonicity when:

if some voters change their mind

to boost the winner, then the winner shouldn't change.

So RCV does <u>not</u> satisfy monotonicity.

## Borda does satisfy monotonicity:

Imagine X is the winner, then we change some ballots to boost X.

Then X will get more points, everybody else either loses pts or stays the same.

Thus X still wins using Borda.

## Explain why plurality satisfies monotonicity.

Imagine X is the winner, and we change ballots to boost X.

Then X gets more rankings in the top

position, and all others stay same or decrease.

Then X still wins using plurality.

---

## A stupid voting system

### Dictatorship

Before the election, one voter is chosen to be the dictator, then everybody votes but only the dictator's vote decides the election.

---

Dict does not satisfy Majority:

even if a majority of voters rank someone 1st, that person may lose.

Unanimity is satisfied by dict.

Imagine all voters rank X above Y.

} then the dictator prefers X above Y,

then Y does not win.

---

Monotonicity

Imagine If X is the winner, then we change ballots to boost X.

Since X won originally, they are the dictator's favorite, so they still win after boosting.

---

IIA is about changing votes.

↑

Independence from Irrelevant Alternatives

idea: If we make "irrelevant" changes to ballots, this should not change the result.

(shuffling around the losers)

**Def** A voting system satisfies IIA when:

If there's a winner, and then we change ballots without moving anyone past the winner, then the winner stays the same.

| 3 | 2 | |
|---|---|---|
| A | B | 2 |
| B | C | 1 |
| C | A | 0 |

using Borda.

A: $3 \cdot 2 + 2 \cdot 0 = 6$

B: $3 \cdot 1 + 2 \cdot 2 = 7$

C: $3 \cdot 0 + 2 \cdot 1 = 2$

B wins

Now change $\begin{matrix} B \\ C \\ A \end{matrix}$ + $\begin{matrix} B \\ A \\ C \end{matrix}$ doesn't move anyone past the winner.

| 3 | 2 | |
|---|---|---|
| A | B | 2 |
| B | A | 1 |
| C | C | 0 |

A: $3 \cdot 2 + 2 \cdot 1 = 8$

B: $3 \cdot 1 + 2 \cdot 2 = 7$

C: $= 0$

A wins!

So Borda does not satisfy IIA.

(swapping the losers can change the winner)

Test Friday covers A1 – A8

A1 - Plurality
A2 - RCV
A3 - Condorcet method
A4 – Borda
A5 - Majority criterion
A6 – Unanimity
A7 – Condorcet Winner crit.
A8 – Monotonicity
A9 - IIA