IIA   Irrelevant Alternatives

Def A system satisfies IIA when: If there is a winner, then we change ballots without moving anyone past the winner. Then the results should be unchanged.

	3	2
2	A	B
1	B	C
0	C	A

using Borda.

$A: 3 \cdot 2 + 0 = 6$

$B: 3 \cdot 1 + 2 \cdot 2 = 7$

$C: 3 \cdot 0 + 2 \cdot 1 = 2$

What if we change $\begin{matrix} B \\ C \\ A \end{matrix} \rightarrow \begin{matrix} B \\ A \\ C \end{matrix}$

Now it's:

	3	2
2	A	B
1	B	A
0	C	C

$A: 3 \cdot 2 + 2 \cdot 1 = 8$

$B: 3 \cdot 1 + 2 \cdot 2 = 7$

$C: \quad 0 \quad = 0$

The winner changed, so we've shown Borda does not satisfy IIA.

$\boxed{\begin{matrix} B \\ C \\ A \end{matrix} \rightarrow \begin{matrix} B \\ A \\ C \end{matrix}}$ ← Counts as "irrelevant" since we didn't move anyone past the winner past the winner.

B, C, A → C, B, A Not allowed for IIA, since we moved C past B.

Test Q: Using RCV,

40	38	20
A	B	C
C	A	B
B	C	A

Make a change showing RCV does not satisfy IIA.

OG winner:

Rd 1

A: 40
B: 38
~~C: 20~~

Rd 2

A: 40
B: 58

B wins

20
C
B
A

can't be changed in IIA

Could do B, A, C → B, C, A but this won't change the winner.

Change

A		C
C	$\longrightarrow$	A
B		B

Now it's:

40	38	20
C	B	C
A	A	B
B	C	A

Rd 1

~~A: 0~~

B: 38

C: 60

Rd 2

B: ~~38~~

C: 60

C wins.

Borda doesn't satisfy IIA

RCV " — — —

also plurality

	IIA
Plurality	✗ ← HW
RCV	✗
Condorcet	✓
Borda	✗
Dictatorship	✓

Condorcet's method <u>does</u> satisfy IIA.

Imagine X is the winner with Condorcet, then we change ballots without moving anyone past X. Then all pairwise results with X will be unchanged, so the results don't change with Condorcet's method.

	Maj.	Unani	CWC	Mono	IIA
Plurality	✓	✓	✗	✓	✗
RCV	✓	✓	✗	✗	✗
Condorcet	✓	✓	✓	✓	✓
Borda	✗	✓	✗	✓	✗
Dictatorship	✗	✓	✗	✓	✓

not usable since it doesn't always choose a winner

Is there a system satisfying all these, which chooses a winner every time?

NO!

"no perfect system exists"

Arrow's Impossibility Theorem (1950s)

The only system satisfying Unanimity & IIA is dictatorship. (Cond. method doesn't count)

Another criterion: Manipulability (A10)

We don't want our system to be "manipulable"

it should be impossible to game the system with some mathy tricks.

"Gaming the system" means voting a way that's different from my true feelings in order to get a better outcome for me.

"Strategic voting"

"Voting manipulation"

We would like for our systems to be non-manipulable.

Plurality is manipulable

Bush-Gore-Nader

2.9	2.9	.1
B	G	N
G	B	G
N	N	B

plurality:
B: 2.9 ← B wins
G: 2.9
N: .1

Can someone change their vote to get a better result (in their own opinion)?

N, G, B → G, N, B, then the result is:

2.9	2.9	.1
B	G	G
G	B	N
N	N	B

B: 2.9
G: 3.0 ← G wins
N: 0

Now G wins, which is a preferable outcome to the N, G, B voters.

Is RCV manipulable? Yes!

6	4	3
A	C	B
B	B	C
C	A	A

using RCV.

Show how some of the voters can manipulate the result.

↑ change their mind to get a better outcome.

Original:

Rd 1
A: 6
~~B: 3~~
C: 4

Rd 2
A: 6
C: 7

A
B 's got their worst result,
C

they should try
B
A
C

6	4	3
B	C	B
A	B	C
C	A	A

Rd 1
~~A: 0~~
B: 9
C: 4

Rd 2
B: 9
C: 4

B wins

B is preferable to the
A
B voters,
C

So they got a better result.

Thm The Gibbard – Satterthwaite Theorem

The only non-manipulable system is dictatorship.

Borda is super-manipulable: insane things happen if everyone tries to vote strategically.

BGN:

	2.9	2.9	.1
2	B	G	N
1	G	B	G
0	N	N	B

Using Borda, the strategy is to dump your most serious rival to the bottom.

Strategic voting would be like:

	2.9	2.9	.1
2	B	G	N
1	N	N	G
0	G	B	B

Results:

B; $2.9 \cdot 2 + 0 + 0 = 5.8$

G: $0 + 2.9 \cdot 2 + .1 = 5.9$

N: $2.9 \cdot 1 + 2.9 \cdot 1 + .1 \cdot 2 = 6.0$

N wins!

(A11) Random Dictator Method

Everybody votes, one ballot is chosen at random, that ballot determines the winner.

Sounds crazy, but works great in some situations.

If X% of voters vote for some outcome, then the probability of that outcome is actually X%.

Works really well for repeated votes.