

## Random Dictator method

Everybody votes, then one vote is chosen at random, their top choice wins.

Defeats "the Tyranny of the Majority"

If 51% of the people agree, in a democracy they get their way 100% of the time.

This was a big concern in the old days.

Using Rand. dictator, if 51% agree, they get their way 51% of the time.

Statistically, this is great.

Works great for repeated elections, with individual low stakes.

One last system:

## Approval Voting

On the ballot, don't rank, but each voter indicates which candidates they approve of.

Check as many as you like.

A   
B   
C   
D

The candidate with the most approvals wins.

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A typical election looks like

	23	37	4	17	21	3
A	x	x		x		x
B	x				x	x
C			x	x		x
D		x			x	x

$$A: 23 + 37 + 17 + 3 = 80$$

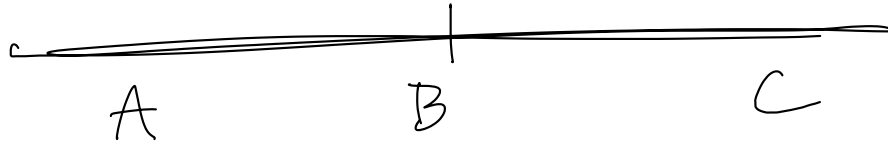
$$C: 4 + 17 + 3 = 24$$

$$B: 23 + 21 + 3 = 47$$

$$D: 37 + 21 + 3 = 61$$

A wins!

This tends to elect middle-ground candidates.



# Weighted Voting

Electoral College system

Each state gets a certain amount of points

"weights" 

CA gets 54

TX 40

FL 30

NY 28

⋮

DE 3

To be the president,  
you need to get the  
most electoral college  
points.

This is a weighted voting system.

A system in which different voters  
count for different amounts.

Used in corporate voting for  
public companies.

Terms: Each voter has a weight

We'll focus on voting for yes/no choices.

There is always a quota ← total amount needed to win.

(not always 50%)

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Ex 5 voters, weights are 12, 10, 5, 1, 1,  
with quota of 20.

Notation:

$[ 20 : 12, 10, 5, 1, 1 ]$   
↑  
quota  
weights

In this system, who has  
the power? Answers are nonobvious!

Here, to get to 20, we must use the 12 and 10.

so 12 & 10 here have equal power,

5, 1, 1 all have no power.

The weights don't really indicate the different levels of power.

Big question: How can we measure the actual power?

Ex [30 : 10, 10, 10, 9]

Here, the 9 is irrelevant.

There are some restrictions on the quota.

[20 : 1, 1, 1]

quota is too big, bigger than the total sum of the weights.

We always require:

$$q \leq w_1 + w_2 + \dots + w_N$$

[2 : 5, 5, 4, 1] <sup>q</sup> was too small. also not allowed because you can meet the quota

even with majority opposed.

We require:

$$q > \frac{1}{2} (w_1 + \dots + w_N)$$

All together:

$$\frac{1}{2} (w_1 + \dots + w_N) < q \leq w_1 + \dots + w_N$$

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Basic measures of power

$$[10 : 12, 4, 2, 2]$$

↑  
12 meets the quota all by himself.

the 12 has all the power.

We say the 12 is a dictator

easy to spot: the weight meets or exceeds  
the quota by itself.

A voter with no power at all is  
called a dummy

$[30: 10, 10, 10, 9]$

9 is a dummy, no dictator.

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$[10: 5, 4, 1, 1]$

dict? NO

dum? NO

veto? 5 & 4

← even the 1 can make  
a difference:  $5+4+1$

The 5 & 4 are not dictators, but  
they are necessary in order to reach the  
quota.

We say they have veto power