

Shapley - Shubik is about voting perms

$$[12 : 8, 6, 4, 2]$$

perms

C B A D

4 6 8 2

pivotal

A.

Dicts & Dummies in SS.

If there is a dictator: the dict. will be pivotal every time, nobody else is ever pivotal.

so the SS score will be

dict: 100%.

all others: 0%.

If there is a dummy, their SS score will be 0%.

Another power index:

Banzhaf Power Index

Another measure of power in weighted voting.

Measures in a different way, so answers are

different from SS.

Banzhaf was a lawyer in Nassau County

6 weighted districts:

Hempstead #1 : 31

Hempstead #2 : 31

N. Hempstead : 28

Oyster Bay : 21

Glen Cove : 2

Long Beach : 2

need 58 votes
to pass a new law.

Banzhaf realized the 21, 2, 2 are all dummies.

about 22% of the people had NO possible
voting power.

B. invented the power index to prove he
was disenfranchised

in 1993 NY outlawed weighted voting.

Banzhaf

Consider all possible combinations of voters who might vote "yes" together.

A B C
[16 : 12, 10, 5]

Combos	total weight
A B C	27
A B	22
A C	17
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B C	15
A B C	12
B	10
C	5
nobody $\rightarrow \emptyset$	0

Only consider the combos which reach the quota.

In each combo, we decide who is critical

A voter is critical in some combo if their vote was necessary in order to meet the quota. [16 : 12, 10, 5]

in ABC ^{total} 27, who is critical?

C is not: without C, A & B still makes it to the quota.

B is not: . . . - - - - -

A is critical: without A, B&C makes 15,
which isn't enough.

[16:12,10,5]

in AB : 22 who is critical?

A is critical

B is also critical.

in AC : 17, A&C are both critical.

In 2016: Trump vs Clinton
↑
winner

Trump's states are a combo.

which are critical? Only TX is critical.

in 2020 which states are critical for Biden?

CA is critical, no others.

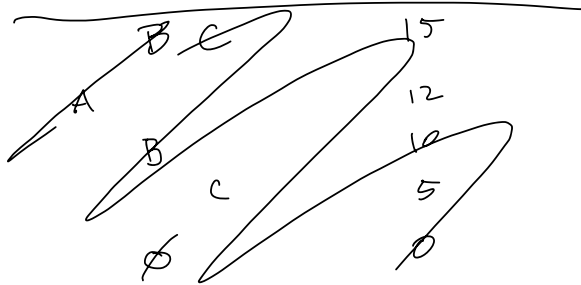
in 2000, Bush won.

Which states are critical for Bush?

All Bush were critical.

$$[16:12, 10, 5]$$

<u>combos</u>	<u>total</u>	<u>critical</u>		
		<u>A</u>	<u>B</u>	<u>C</u>
A B C	27	✓		
A B	22	✓	✓	
A C	17	✓		✓



Final answer :

$$A: \frac{3}{5} = 60\%$$

$$B: \frac{1}{5} = 20\%$$

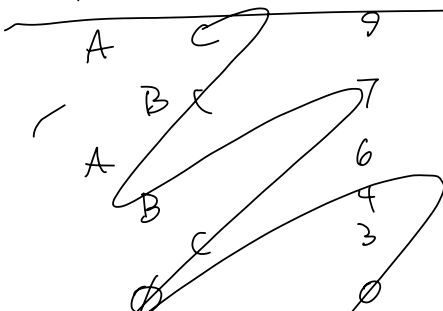
$$C: \frac{1}{5} = 20\%$$

Each fraction is :

$$\frac{\# \text{ of times } X \text{ is critical}}{\text{total } \# \text{ of critical } \checkmarks}$$

Find the Banhof of $[10:6, 4, 3]$

<u>combos</u>	<u>total</u>	<u>A</u>	<u>B</u>	<u>C</u>
A B C	13	✓	✓	
A B	10	✓	✓	

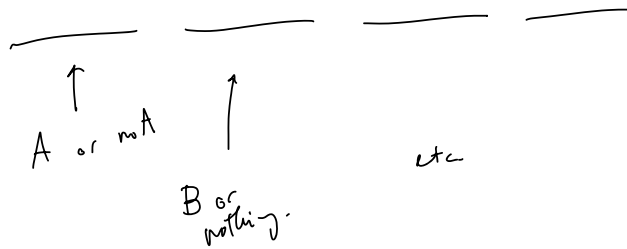


A: $\frac{2}{4}$
 B: $\frac{2}{4}$
 C: $\frac{0}{4}$

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How many combos will there be?

For ABC, there's 8.

for ABCD :



each slot has 2 possibilities.

Total # of options is $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

For N voters, there are 2^N different
combos.

Try Banzhaf for
 $\begin{matrix} & A & B & C & D \\ \text{crits} & 5 & 4 & 3 & 1 \end{matrix}$
 $[8: 5, 4, 3, 1]$

combos	total	A	B	C	D
A B C D	13	✓			
A B C	12	✓	✓		
A B D	10	✓			✓
A C D	9	✓			✓
B C D	8		✓	✓	✓
A B	9	✓	✓		
A C	8	✓		✓	
<hr/>					
A D	7				
B C	⋮				
B D	⋮				
C D					
A					
B					
C					
D					
∅					

A: $5/12$ B: $3/12$
 C: $3/12$ D: $1/12$

What about dict/dum in Banzhaf?

If there is a dictator, they are critical every time, and nobody else is ever critical, so the Banzhaf will say:

Dictator : 100%

others : 0%

Any dummy will never be critical,
so a dummy automatically has

Banzhaf = 0 %