New test topics

B3 - Banzhaf

C1 ← determine possible outcomes

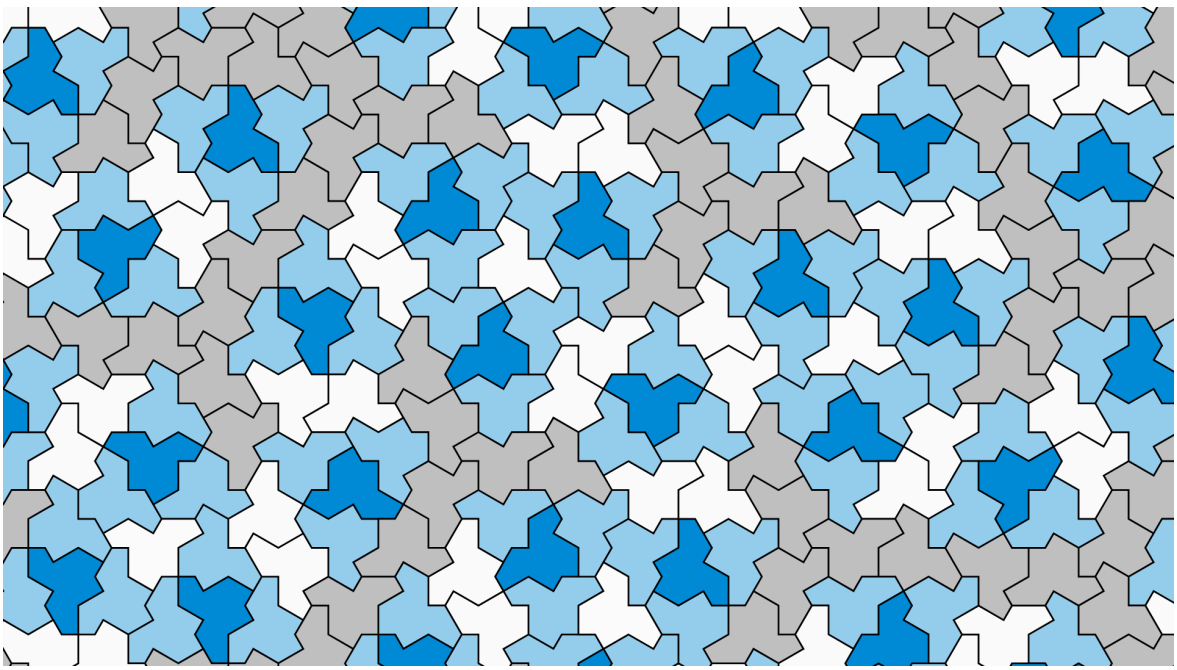
C2 draw districts

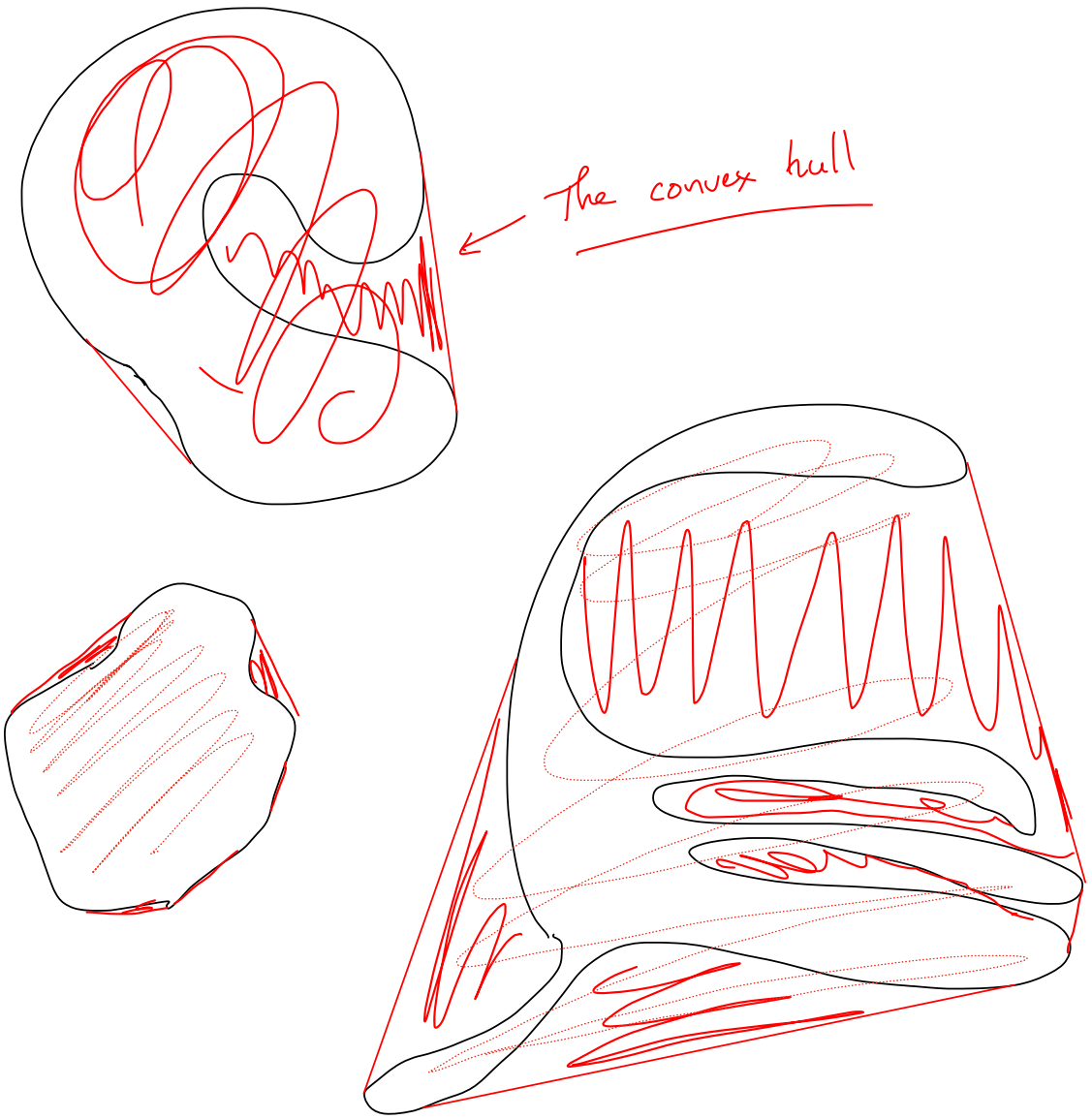
C3 efficiency gap

C4 CH ratio

C5 IQ

Aperiodic tilings





A "normal" shape is similar to its own CH.

A weird wormy shape has CH much bigger than the OG shape.

(by area)

The Convex hull Ratio (CHR)

is $CHR = \frac{\text{area of original shape}}{\text{area of CH shape.}}$

If the shape is nice, OG area is almost the same as CH area,

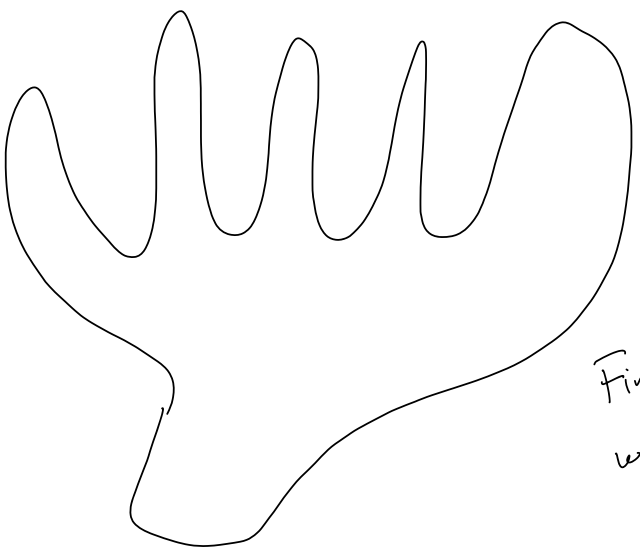
so CHR is close to $1 = 100\%$ for a "nice" shape.

For a weird shape, CH area is much bigger, so the CHR will be close to 0.

CHR is like a "score" for how weird the shape is.

0 → very weird

1 → not weird.



$CHR = \frac{\text{OG area}}{\text{CH area.}}$

Finding area of a weird shape is hard.

The fabulous dot planimeter

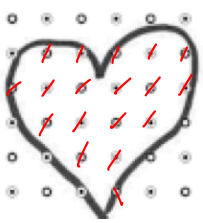
Finding the area of a weird shape is hard! So we superimpose dots on the picture, separated by 1cm (or whatever). Count the dots that lie inside the shape.

Counting the dots is like counting the squares, so gives a pretty good estimate of the area. Looks like 17 cm^2 (or whatever).

What if it's on the line?

A dot on the line represents an area of 1/2. So those should be counted as half. Or equivalently, only count up half of them.

- If the dot isn't on the line, count it no matter what.
- If the dot is on the line and looks like ⊙, count it.
- If the dot is on the line and looks like ⦿, don't count it.



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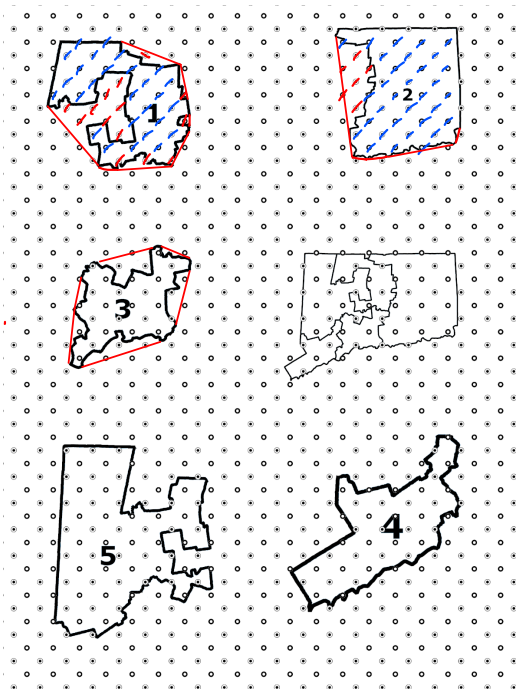


#1: OG: 28 CH: 42 CHR = $\frac{28}{42}$ = .66 66%.

#2: OG = 32 CH = 38 CHR = $\frac{32}{38}$ = 84%.

#3: $\frac{20}{31}$ CT: $\frac{44}{50}$

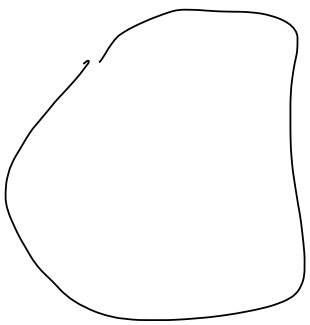
#5 $\frac{52}{70}$ #4: $\frac{36}{49}$

Isoperimetric Quotient. IQ

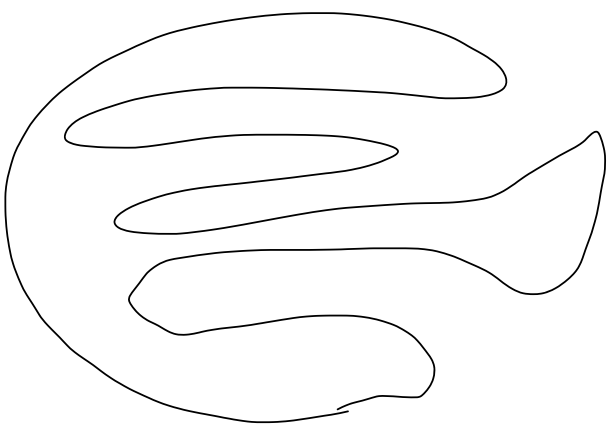
A different "score" for weird shapes

IQ close to 0 is bad

IQ close to 1 (100%) is good.



nice perimeter is not so big



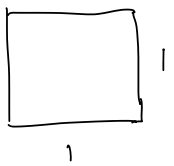
weird

big perimeter, not much area.

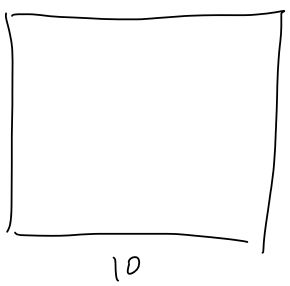
Isoperimetric Quotient

measures Area vs perimeter.

basically $\frac{A}{P}$, but with some modifications



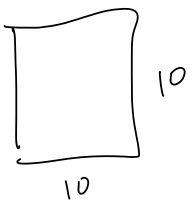
here, $A = 1$, $P = 4$ so $\frac{A}{P} = \frac{1}{4}$



$A = 100$, $P = 40$ so $\frac{A}{P} = \frac{100}{40} = \frac{10}{4} = 2.5$

Instead, use $\boxed{\frac{A}{P^2}}$

$A = 1$, $P = 4$, $P^2 = 16$ $\frac{A}{P^2} = \frac{1}{16}$



$A = 100$, $P = 40$, $P^2 = 1600$ $\frac{A}{P^2} = \frac{100}{1600} = \frac{1}{16}$

The IQ is

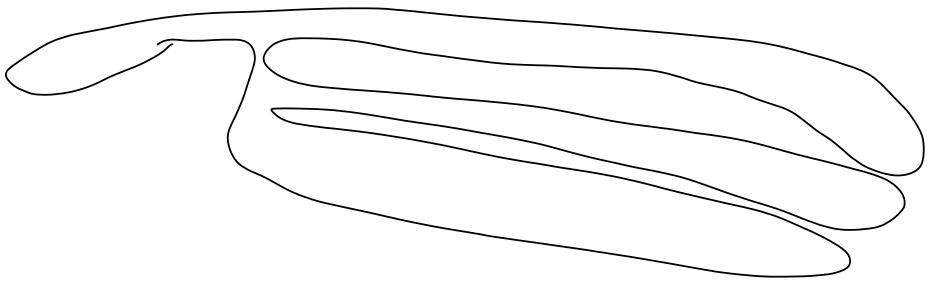
$$IQ = \frac{4\pi A}{p^2}$$

(4π is so that IQ of a circle is 1)

According to IQ,

the best possible shape is a circle.

worst shapes would be long skinny worms



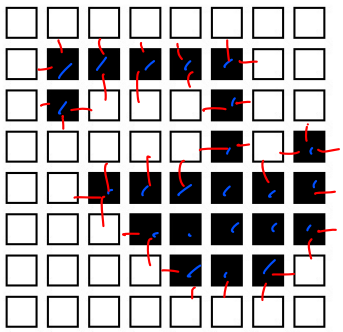
This is also called the

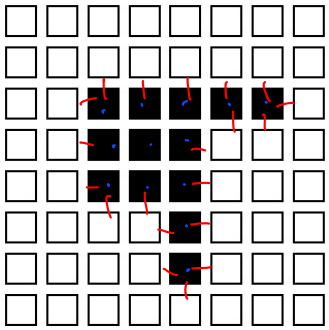
Polsby - Popper score.

$A = 23$

$P = 36$

$IQ = \frac{4\pi \cdot 23}{36^2}$





$A = 13$

$P = 20$

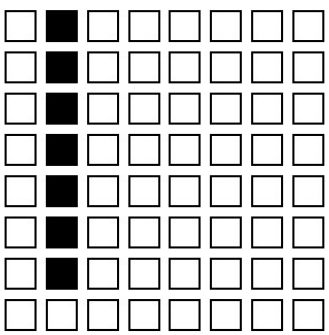
$IQ = \frac{4\pi \cdot 13}{20^2}$

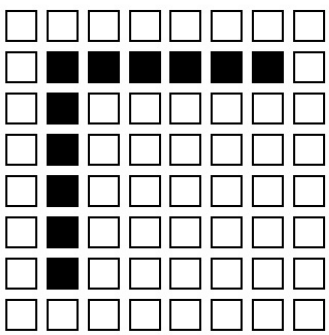
$A = 7$

$P = 16$

$IQ = \frac{4\pi \cdot 7}{16^2}$

$\approx 34\%$



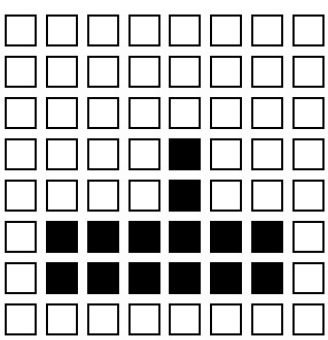


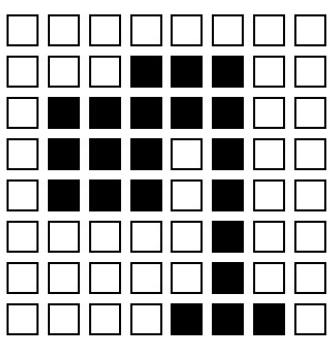
$A = 11$

$P = 24$

$A = 14$

$P = 20$





$A = 21$

$P = 32$

has IQ ≈ 34.

What's the CHR?

$$\frac{\text{OG area}}{\text{CH area}} = 1$$

100%!

IQ & CHR are 2 different ways to measure how weird it is.