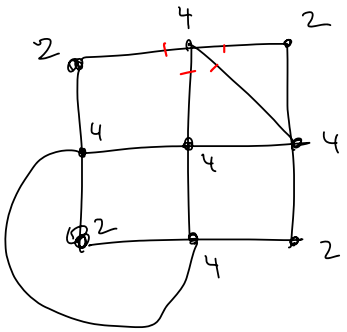
Euler circs/paths

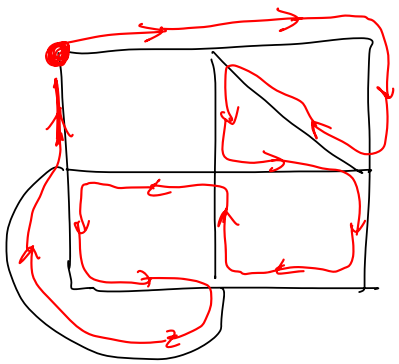
Use every edge, no repeated edges.

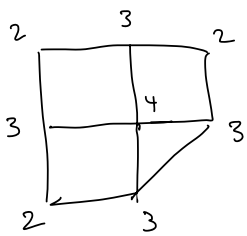


For a E. circ: all degrees must be even

E. path: 2 odds, the rest even.

↖ There is an E. circuit.





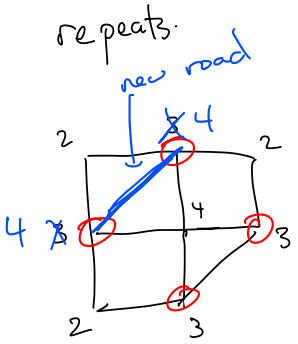
A street grid.

Can Mr Plow drive down each street 1 time without re-using any streets?

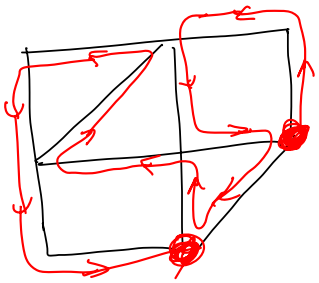
Many odds, so this is impossible.

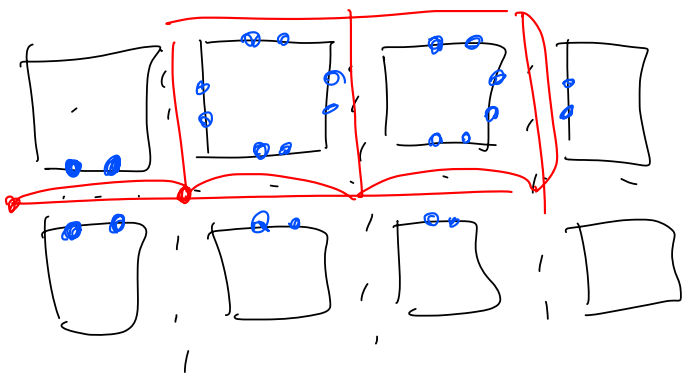
Ms Road wants to build a new road

So that MP can do it with no repeats.



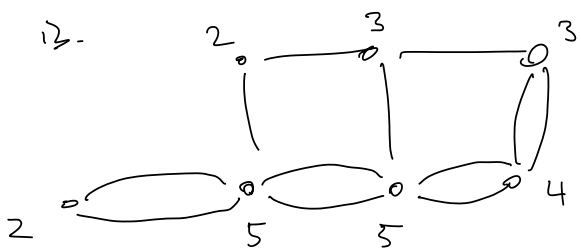
← Now it does have an Euler path.





Can we visit all the dots without extra walking?

The graph is.

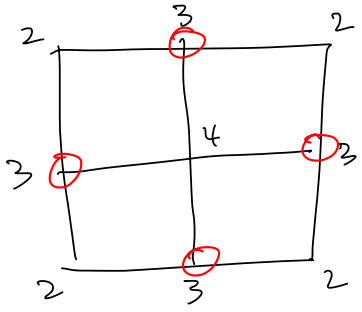


Many odds, so we will need to repeat some edges.

Minimum Duplication Circuits (Paths)

When an Euler circuit doesn't exist, some edges must be repeated.

A minimum duplication circuit is one which uses all edges, repeating as few as possible.

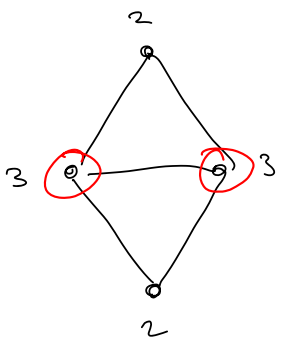


Can we cover all edges with no repeats? NO (there's 4 odds)

What's the smallest amount of repeated edges which would make it possible.

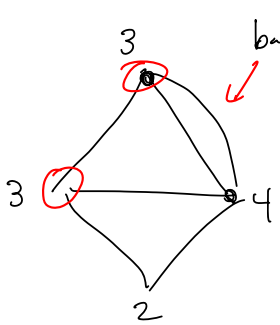
(In this one, we require 4 edge duplications)

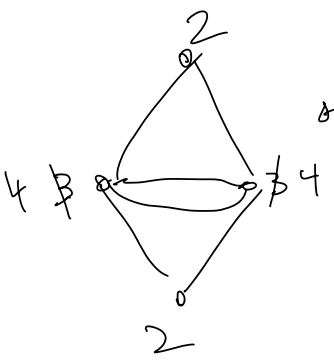
Ex



No Euler circuit (2 odds)

What's the "best" edge to duplicate?

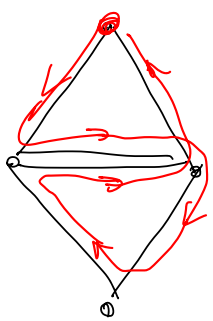




Now they're all even!

Best to duplicate an edge connecting 2 odds.

make an Euler circuit

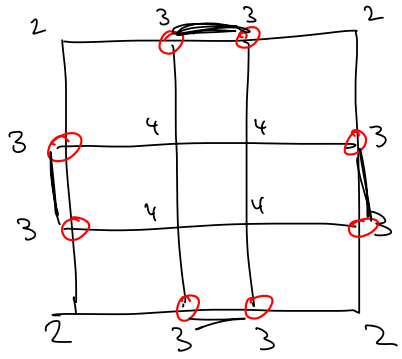


This is a min. dup. circ. using 1 duplicated edge.

To find a min. dup. circ:

- Make all degrees even by doubling edges.
  (Best to double an edge between 2 odds)

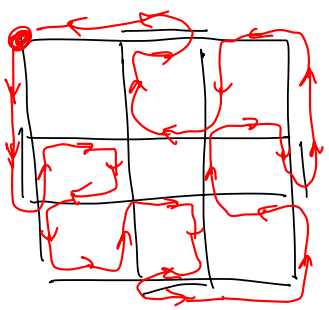
- Draw an Euler circ. on the new graph; this represents a min. dup. circ on the OG.

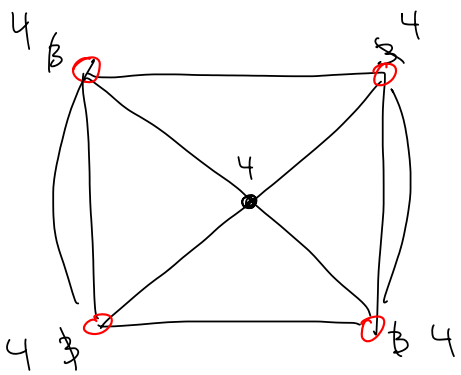


Find a min dup. circ.

try to dupe edges connecting the odds in pairs.

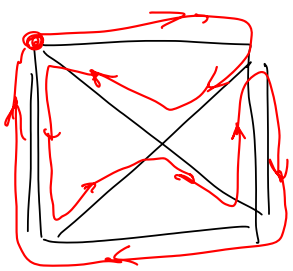
this requires 4 dupes.

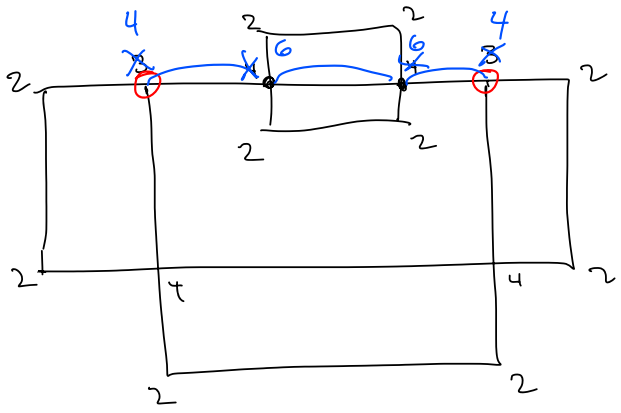




Find a min dup. circ.

Will need 2 dupes.



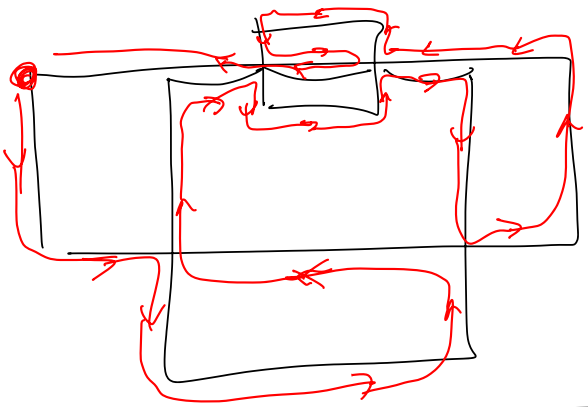


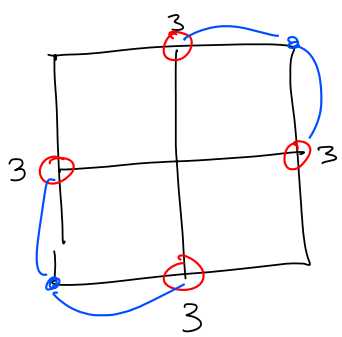
Find a min dup circ.

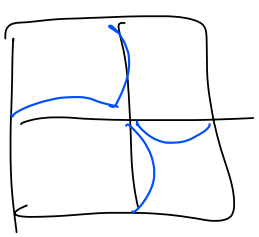
The odds are not connected by an edge, so we need more than 1 dupe.

Find the shortest path connecting the odds, duplicate all edges along that path.

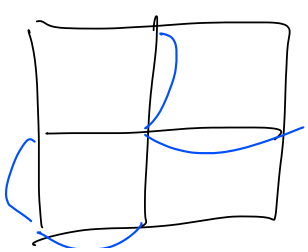
Here we use 3 duplications:







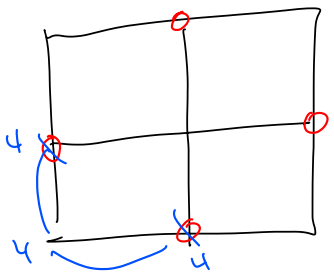
Needs 4 dupes.



Min Dup Paths ← (not circ.)

Make a path with as few dupes as possible.

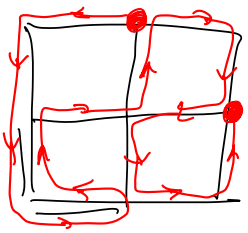
Just like a Min dup circ, but we can leave 2 odds, make the rest even.

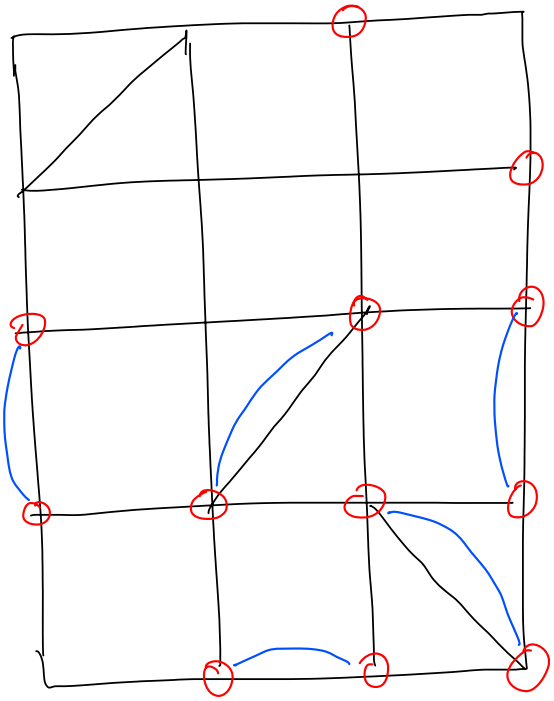


Make a Min dup path:

I need to make all but 2 odds.

My Min. Dup. path looks like:





How many dupes are needed & where?

to get a min dup path?

leave 2 odd,

connect up the rest.

We need 5 dupes.