Math 3342 Exam #1 practice questions

Question 1. For this whole page, let L be this language:

$$L = \{bax \in \{a,b\}^* \mid x = a^n \text{ where } n \text{ is even, or } x = b^n \text{ where } n \text{ is odd}\}$$

a) Please give a DFA or NFA whose language is L.

b) Please give a regular expression which generates L.

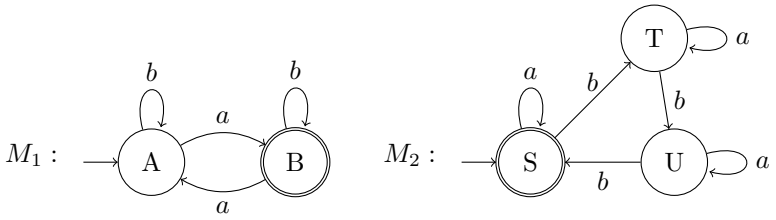
c) Is L a regular language? Please say why or why not.

Question 2. For this whole question, let $L = \{bxb \mid x \in \{a,b\}^*\}$.

a) Please give a DFA for L.

b) Please give an NFA for L using fewer states than your DFA used.

Question 3. Here are 2 DFAs M_1 and M_2.

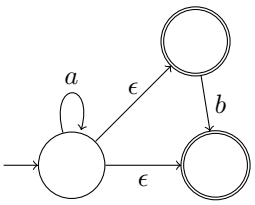


a) Please give regular expressions for each of $L(M_1)$ and $L(M_2)$.

b) Please give a single DFA whose language is $L(M_1) \cup L(M_2)$.

c) Give a DFA whose language is $L(M_1) \cap L(M_2)$.

Question 4. Here is an NFA called N:



a) Please find $L(N)$ and write it as a set, or describe it using ordinary words.

b) Please give a regular expression for $L(N)$.

c) Is $L(N)$ a regular language? Say why or why not.

d) Please convert N to a DFA using the subset construction.

Question 5. Please convert this regular expression to NFA:

$$(ba(a + b^2)^* a)^*$$

Question 6. Find the derivative $\frac{d}{da}$ for each of the following languages:

a) $\{a^n b^n\}$

b) $\{abx \mid x \in \{a, b\}^*\}$

c) $\{bax \mid x \in \{a, b\}^*\}$

d) $\{x \in \{a, b\}^* \mid x \text{ has even length}\}$

Question 7. Consider this NFA:

$$M = (\{A, B, C, D\}, \{a, b\}, \delta, B, \{C, D\})$$

$$\begin{array}{lll} \delta(A, a) = \{B, C\} & \delta(A, b) = \{C\} & \delta(A, \varepsilon) = \emptyset \\ \delta(B, a) = \{D\} & \delta(B, b) = \emptyset & \delta(B, \varepsilon) = \{A\} \\ \delta(C, a) = \{D\} & \delta(C, b) = \{B\} & \delta(C, \varepsilon) = \emptyset \\ \delta(D, a) = \emptyset & \delta(D, b) = \emptyset & \delta(D, \varepsilon) = \{B\} \end{array}$$

a) Please draw the diagram for M.

b) Is $abba$ accepted or rejected? Please either describe a path through the diagram which makes it accepted, or say why it's rejected.

c) Please compute each of these (don't forget about those εs!):

i) $\delta^*(A, b^2)$

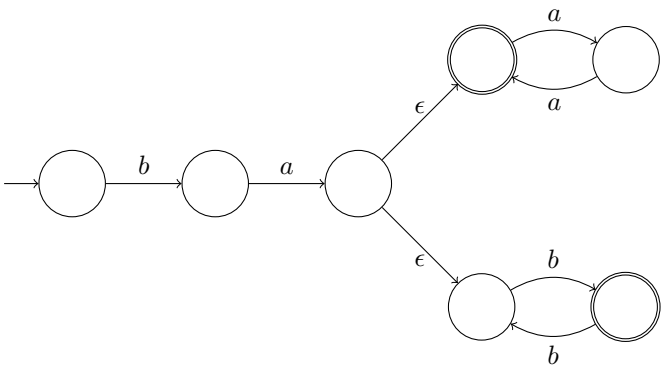
ii) $\delta^*(A, a^2)$

iii) $\delta^*(C, ab)$

d) Please give a string x (other than those mentioned above) which is rejected by N, or say that there are none.

Answers!

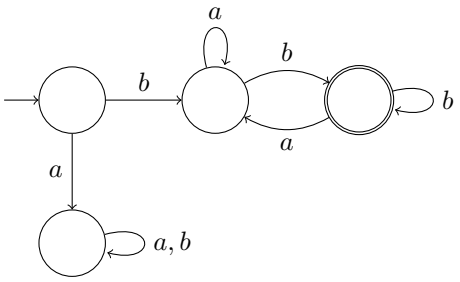
1. a)



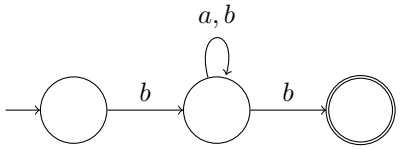
b) $ba((aa)^* + b(bb)^*)$

c) Yes it is: I made an NFA and regex for it, so it is automatically regular.

2. a)

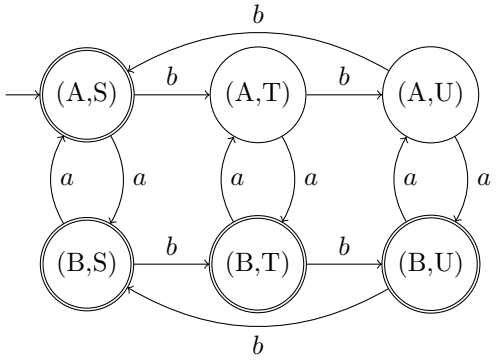


b)

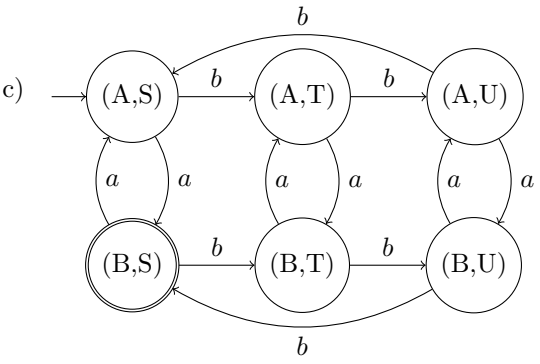


3. a) For M_1: $b^*a(b^*ab^*ab^*)^*$, and for M_2: $a^*(a^*ba^*ba^*ba^*)^*a^*$.

b)



c)

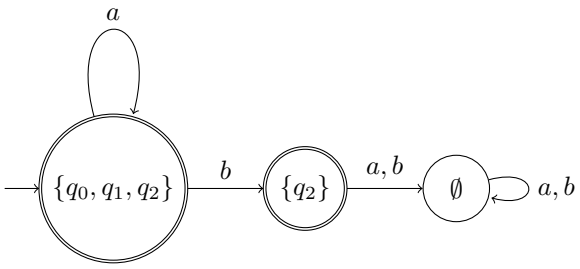


4. a) $L(N)$ is all strings of the form a^n or $a^n b$.

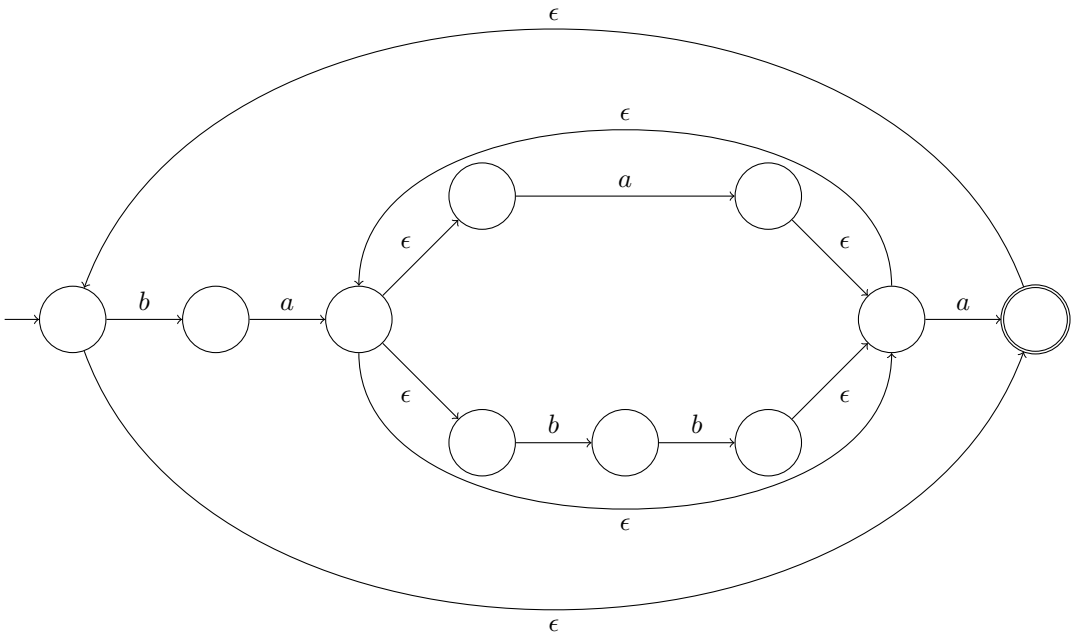
   b) $a^*(\epsilon + b)$ or equivalently $a^*b + a^*$

   c) Yes it is regular, since there is an NFA and a regex for it.

   d) (I called the start state q_0, the upper accepting state q_1, and the other one q_2)



5.



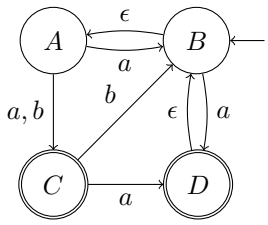
6. a) $\{a^{n-1}b^n\}$

b) $\{bx \mid x \in \{a, b\}^*\}$

c) $\emptyset$

d) $\{x \in \{a, b\}^* \mid x \text{ has odd length}\}$

7. a)



b) $abba$ is accepted. Starting at B, follow arrows like: $a, \epsilon, \epsilon, b, b, a$ and we end up at state D, which is accepting.

c) i) $\{A, B\}$

c) ii) $\{A, B, C, D\}$

c) iii) $\{C\}$

d) ϵ is rejected, also any b^{2n}, probably some others.