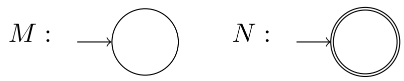
Math 3342 Exam #3 practice

These are sample questions, but this is not meant to be exhaustive of everything that might be on the final exam.

Question 1. Please make a Turing machine that accepts any string of the form axa for $x \in \{a, b\}^*$, and runs forever when the input is not of this form.

Question 2. Let M and N be these machines using the alphabet $\Sigma = \{a, b\}$:



(These each are machines of a single starting state with no transition arrows.)

a) Is M legal when interpreted as a DFA? If so, say what $L(M)$ is. If not, say why not.

b) Is N legal when interpreted as a DFA? If so, say what $L(N)$ is. If not, say why not.

c) Is M legal when interpreted as a NFA? If so, say what $L(M)$ is. If not, say why not.

d) Is N legal when interpreted as a NFA? If so, say what $L(N)$ is. If not, say why not.

e) Is M legal when interpreted as a Turing Machine? If so, say what $L(M)$ is. If not, say why not.

f) Is N legal when interpreted as a Turing Machine? If so, say what $L(N)$ is. If not, say why not.

Question 3. For this problem, let G be this grammar:

$$\begin{aligned} S &\to aTX \\ T &\to bR \\ R &\to S \mid \varepsilon \\ X &\to aX \mid \varepsilon \end{aligned}$$

a) Please give a grammar derivation for some nonempty string in $L(G)$.

b) Please describe in words or set theory notation the set $L(G)$.

c) Please give a regular expression equivalent to G, or explain why this is not possible.

d) Please give a stack machine equivalent to G, and show a stack derivation of some specific nonempty string.

Question 4. Please make a Turing machine with language $\{a^{2n}cb^n\}$.

Question 5. Please make a Turing machine that computes this function: $f(a^n) = a^{n \bmod 2}$. (The answer left on the tape should be either a or ε, depending if n is even or odd.)

Question 6. Please give an NFA for this regular expression: $(a + b)^*ab + a^*b$

Question 7. In each part, either show that the language is not regular, or show that it is regular.

a) $\{a^n bxab^n \mid x \in \{a, b\}^*\}$

b) $\{a^n xb^m \mid x \in \{a, b\}^*\}$

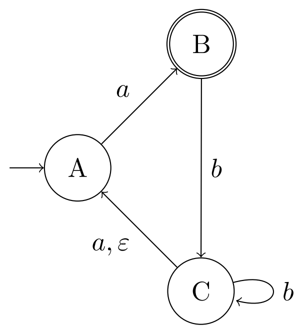
Question 8. In each part, please give a grammar for each language:

a) $\{ab^n a\}$

b) $\{a^n b^m c^{2m}\}$

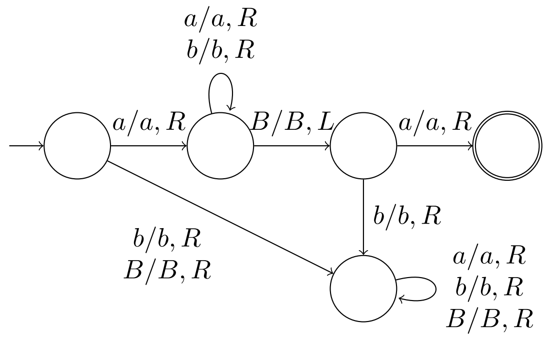
c) $\{a^n x(ab)^n \mid x \in \{a, b\}^*\}$

Question 9. Please convert this NFA to an equivalent DFA:



Answers!

1.



2. a) M is not a legal DFA, since it would need outgoing arrows for each letter in the alphabet.
   b) N is not a legal DFA for the same reason.
   c) M is a legal NFA. Its language is $L(M) = \emptyset$.
   d) N is a legal NFA. Its language is $L(N) = \{\varepsilon\}$.
   e) M is a legal Turing Machine, with $L(M) = \emptyset$.
   f) N is a legal Turing Machine, with $L(N) = \{a, b\}^*$.

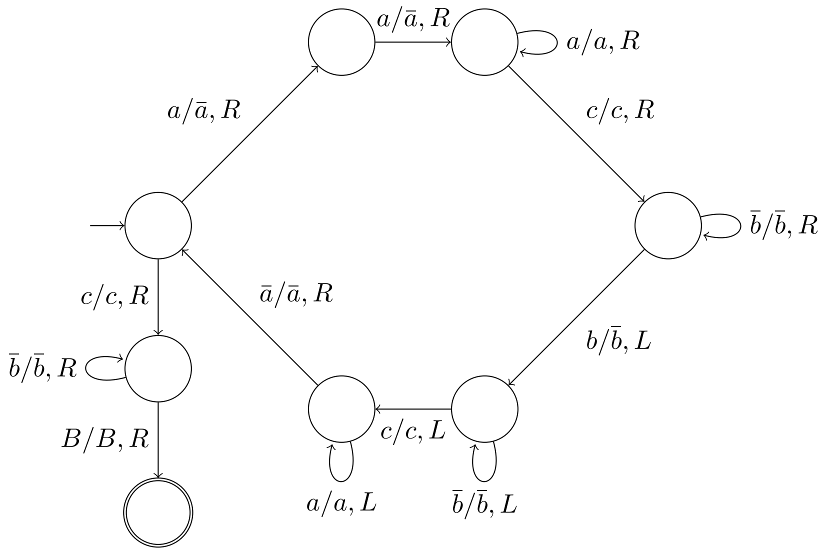
3. a) I'll derive ab: $S \Rightarrow aTX \Rightarrow abRX \Rightarrow abX \Rightarrow ab$
   b) $L(G) = \{(ab)^n a^m \mid n \geq 1, m \geq 0\}$
   c) $ab(ab)^*a^*$
   d) My stack machine:

read	pop	push
ϵ	S	aTX
ϵ	T	bR
ϵ	R	S
ϵ	R	ϵ
ϵ	X	aX
ϵ	X	ϵ
a	a	ϵ
b	b	ϵ

A derivation of ab:

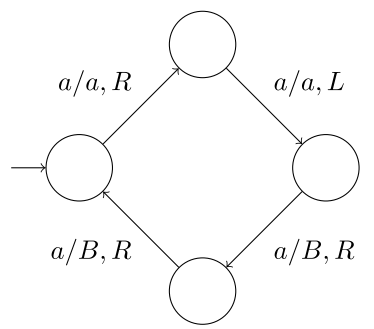
$$(ab, S) \mapsto (ab, aTX) \mapsto (b, TX) \mapsto (b, bRX) \mapsto (\epsilon, RX) \mapsto (\epsilon, X) \mapsto (\epsilon, \epsilon)$$

4. My strategy is to mark 2 as, then go right and mark one b. Make sure not to loop on c/c, since this would allow for any number of c's. We want to require just one.

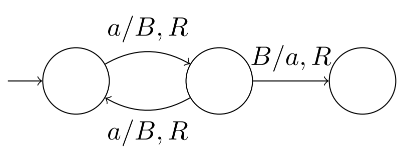


5. This one is a bit tricky. My strategy is: we see an a first, then move R and look for another a. If there isn't one, then we had only one a left, so the string is odd, so leave the a and stop. If we see a second a, then back up and blank both a's. Then repeat.

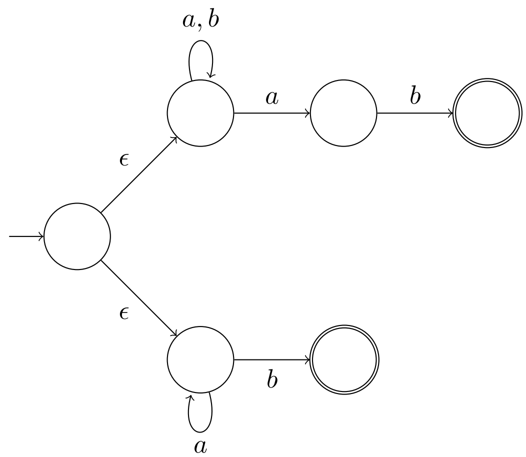
   This ends up blanking 2 as at a time, so we end up with either empty, or one leftover a, depending if n was even or odd.



   Here's another solution which is cute but a bit cryptic:



6.



7. a) This one is not regular: let $D_i = \dfrac{d}{da^i} L = \{a^{n-i}bxab^n \mid x \in \{a, b\}^*\}$. Then these are all different, so L has infinitely many derivatives, so L is not regular.

   b) This one is regular. Here is a regular expression: $a^*(a + b)^*b^*$.

8. a)

$$
\begin{aligned}
S &\to aTa \\
T &\to bT \mid \epsilon
\end{aligned}
$$

   b)

$$
\begin{aligned}
S &\to AT \\
A &\to aA \mid \epsilon \\
T &\to bTcc \mid \epsilon
\end{aligned}
$$

   c)

$$
\begin{aligned}
S &\to aSab \mid T \\
T &\to aT \mid bT \mid \epsilon
\end{aligned}
$$

9.