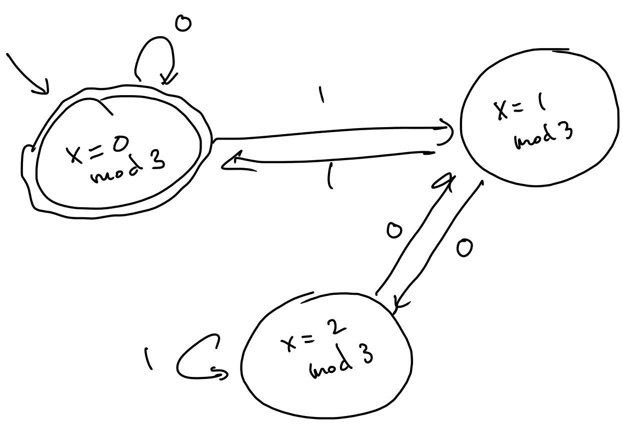
A DFA for binary #s,

accept if the number is div. by 3

reject otherwise.

0	0
1	1
10	2
11	3
100	4
101	5
110	6



101101011100

If we see a # x, then followed by 0

111 vs 1110

↑ ↑

7 14

111 means $1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

Adding a 0 to the right of (binary) x changes it to 2x.

Adding a 1 . – – – – –

. . . – $2x+1$.

If $x=0 \bmod 3$, what about x followed by 0?

$x=3k$ So, it becomes $2(3k) = 3(2k)$

still div. by 3.

if $x=0 \bmod 3$ and we read 1,

$x=3k$ it becomes $2(3k)+1 = 3(2k)+1$

If $x=1 \bmod 3$ and we read 0,

$x=3k+1$ it becomes $2(3k+1) = 6k+2 = 3(2k)+2$

If $x=1 \bmod 3$, . . . – . 1,

$x=3k+1$ becomes $2(3k+1)+1 = 6k+2+1$

$= 6k+3 = 3(2k+1)$

$= 0 \bmod 3$.

If $x=2 \bmod 3$, read a 0

$x=3k+2$ becomes $2(3k+2) = 6k+4$

$= (6k+3)+1$

If $x=2 \bmod 3$, read a 1,

$x=3k+2 \longrightarrow 2(3k+2)+1$

$6k+4+1 = (6k+3)+2$

What is a DFA?

A formal description using sets

Important ingredients:

The states ← set

the transitions ← function

The accepting states ← set.

The starting state ← an element.

The alphabet ← a set.

Def A DFA is a 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

where Q is a finite set (set of states)

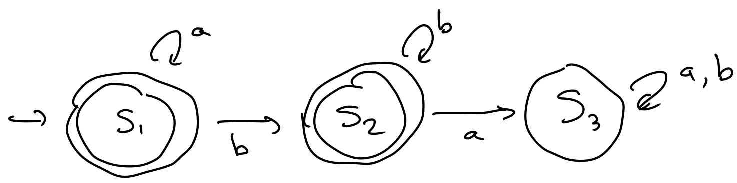
Σ is a finite set (alphabet)

$\delta : Q \times \Sigma \to Q$ is a function (the transition function)

$q_0 \in Q$ (the starting state)

$F \subseteq Q$ (the set of accepting states)

Ex



write this DFA formally as a 5-tuple.

it's $$M = \left(\{S_1, S_2, S_3\}, \{a, b\}, \delta, S_1, \{S_1, S_2\} \right)$$

where δ is: $\delta: Q \times \Sigma \to Q$

$\delta(S_1, a) = S_1$ $\qquad$ $\delta(S_1, b) = S_2$

$\delta(S_2, a) = S_3$ $\qquad$ $\delta(S_2, b) = S_2$

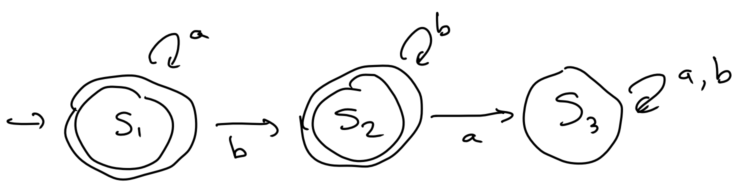
$\delta(S_3, a) = S_3$ $\qquad$ $\delta(S_3, b) = S_3$

We want to plug entire strings into δ.

"the extended trans. function"

$$\delta^*: Q \times \Sigma^* \to Q$$

$\delta^*(S, x) =$ where you end up if you start in S & read the string x.



Here, $\delta^*(S_2, baab) = S_3$.

Properties of δ^*:

- $\delta^*(q, \varepsilon) = q$
- if $|x| = 1$. i.e. x is a single letter

$$\delta^*(q, x) = \delta(q, x)$$

- $\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$

<u>Def</u> For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

we say a string $x \in \Sigma^*$ is <u>accepted</u> <u>by</u> $\underline{M}$

when: $\delta^*(q_0, x) \in F$

$$L(M) = \{ x \in \Sigma^* \mid \delta^*(q_0, x) \in F \}$$