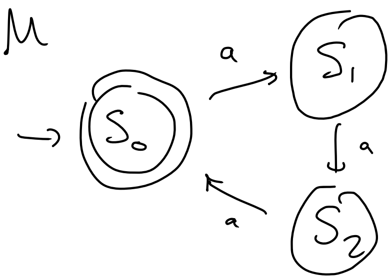
M



This DFA accepts any $x \in \{a\}^*$ with $|x|$ div. by 3.

Prove this using the formal description

Let's prove $L(M) = \{a^n \mid n \equiv 0 \mod 3\}$.

Formally, $M = (Q, \Sigma, \delta, q_0, F)$

where $Q = \{S_0, S_1, S_2\}$

$\Sigma = \{a\}$

$\delta(S_0, a) = S_1$

$\delta(S_1, a) = S_2$

$\delta(S_2, a) = S_0$

$q_0 = S_0$

$F = \{S_0\}$

and $L(M) = \{x \in \Sigma^* \mid \delta^*(q_0, x) \in F\}$

$= \{x \in \{a\}^* \mid \delta^*(S_0, x) = S_0\}$

want to show

WTS this equals $\{a^n \mid n \equiv 0 \mod 3\}$.

Need to describe δ^* in convenient terms

$\delta(S_0, a) = S_1$

$\delta(S_1, a) = S_2$

$\delta(S_2, a) = S_0$

$\delta(S_i, a) = S_{i+1} \pmod 3$

$\delta^*(S_i, a) = S_{i+1} \pmod 3$

$\delta^*(S_i, a^2) = S_{i+2} \mod 3$

$\delta^*(S_i, a^n) = S_{i+n} \pmod 3$

$$\overset{L(M)}{S_0} = \{ x \mid \delta^*(S_0, x) = S_0 \}$$

$$= \{ a^n \mid \delta^*(S_0, a^n) = S_0 \}$$

$$= \{ a^n \mid S_n = S_0 \mod 3 \}$$

$$= \{ a^n \mid n = 0 \mod 3 \}$$

<u>Shown</u>

A lang which can be accepted by DFA is a <u>regular language</u>

What happens when we combine reg. langs?

If L_1 & L_2 are regular,

is $L_1 \cup L_2$ regular?

or $L_1 \cap L_2$? etc.

These are "closure properties"

Are reg. langs closed under $\cup$?
$\cap$?

Closure under complement

If L is a language on Σ^*,

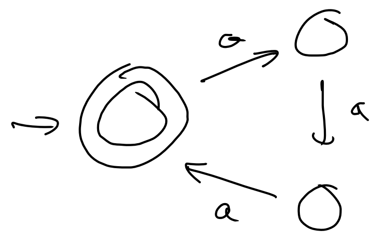
then $\overline{L} = \Sigma^* - L$ (all strings not in L)

is the complement

If L is regular, then $\overline{L}$ is regular.

if we have a DFA accepting L,

can we build a DFA which accepts $\overline{L}$?



accepts $L = \{a^n \mid n = 0 \bmod 3\}$

a machine for $\overline{L} = \{a^n \mid n \neq 0 \mod 3\}$

Formally: If $M = (Q, \Sigma, \delta, q_0, F)$,

with $L(M)$ some language,

which DFA accepts $\overline{L(M)}$?

It should be

$$M' = (Q, \Sigma, \delta, q_0, \overline{F})$$

Let's prove $L(M') = \overline{L(M)}$

$$L(M') = \{x \in \Sigma^* \mid \delta^*(q_0, x) \in \overline{F}\}$$

$$= \{x \in \Sigma^* \mid \delta^*(q_0, x) \notin F\}$$

$$= \{x \mid x \notin L(M)\}$$

$$= \overline{L(M)}$$

So

Thm If L is regular, then $\bar{L}$ is regular.

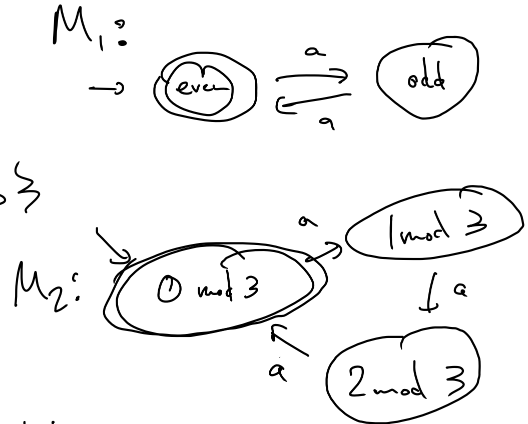
"reg langs are closed under complements"

What about $\cap$?

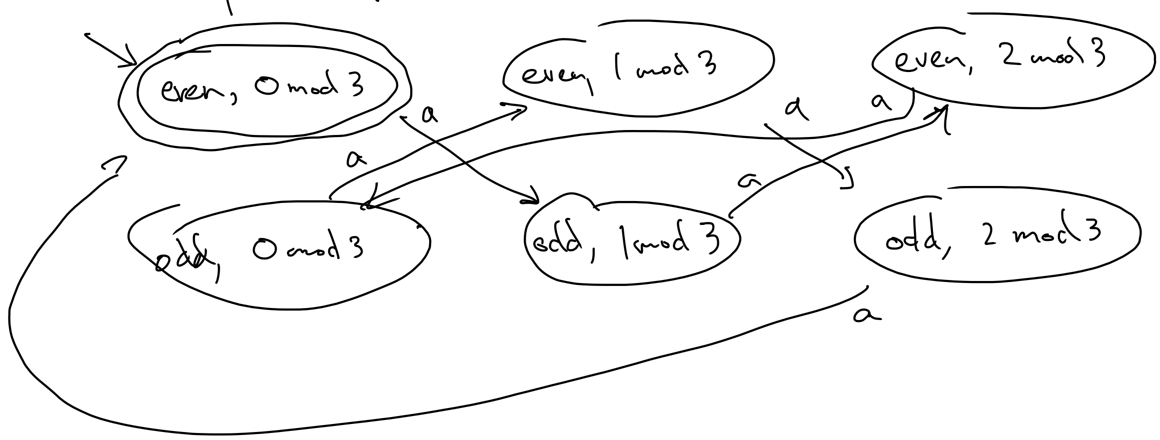
Thm If L_1 & L_2 are regular, then $L_1 \cap L_2$ is regular.

Idea: $L_1 = \{ a^n \mid n \text{ is even} \}$

$L_2 = \{ a^n \mid n \text{ is div by } 3 \}$



Create a new machine where states are pairs from M_1 & M_2



Make pairs of states, connect arrows according to where each part of the pair goes

Start state is the pair of the original start states,

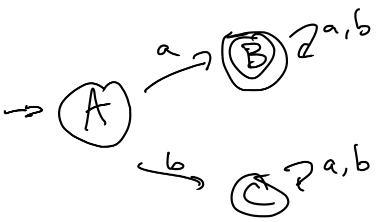
Accepting states are all where <u>both</u> parts of the pair are accepting.

Called "The product construction"

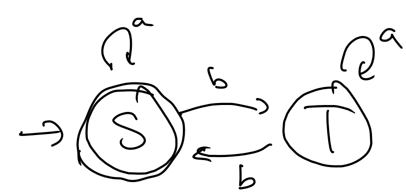
$L_1 = \{ ax \mid x \in \{a,b\}^* \}$

$L_2 = \{ x \in \{a,b\}^* \mid x \text{ uses an even \# of } b\text{'s} \}$

M_1 :



M_2 :



start with the starting state, draw the diagram from there.