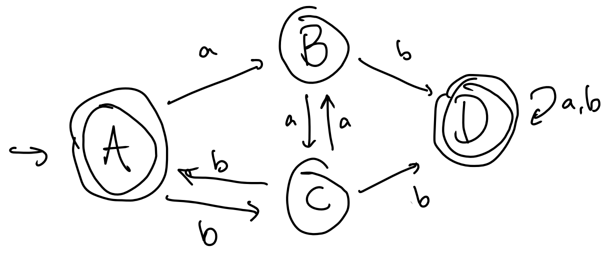
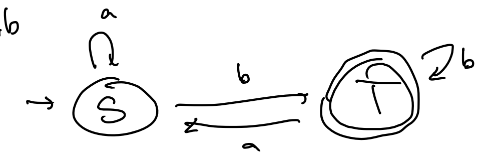
Intersection of regular langs

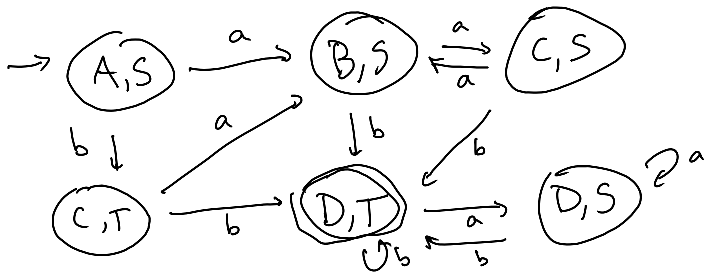


M_1



M_2

The product construction:



(A,T would also be accepting, but we didn't use it)

What does it look like formally?

$$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$$

$$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$$

Then the product construction creates:

$$M_3 = (Q \times R, \Sigma, \delta, (q_0, r_0), F_1 \times F_2)$$

where $\delta((q,r), a) = (\delta_1(q,a), \delta_2(r,a))$

Let's prove $L(M_3) = L(M_1) \cap L(M_2)$

Rember $L(M) = \{x \in \Sigma^* \mid \delta^*(q_0, x) \in F\}$

<u>Thm</u> If M_3 is the "product machine" as above of M_1 & M_2, then $L(M_3) = L(M_1) \cap L(M_2)$

<u>Pf</u>
$$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$$
$$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$$
$$M_3 = (Q \times R, \Sigma, \delta, (q_0, r_0), F_1 \times F_2)$$
where $\delta((q,r),a) = (\delta_1(q,a), \delta_2(r,a))$

$$L(M_3) = \{x \in \Sigma^* \mid \delta^*((q_0, r_0), x) \in F_1 \times F_2\}$$
$$= \{x \in \Sigma^* \mid \underbrace{(\delta_1^*(q_0,x), \delta_2^*(r_0,x))}_{} \in \underbrace{F_1 \times F_2}_{}\}$$
$$= \{x \in \Sigma^* \mid \delta_1^*(q_0,x) \in F_1 \text{ and } \delta_2^*(r_0,x) \in F_2\}$$
$$= \{x \in \Sigma^* \mid \delta_1^*(q_0,x) \in F_1\} \cap \{x \in \Sigma^* \mid \delta_2^*(r_0,x) \in F_2\}$$
$$= L(M_1) \cap L(M_2)$$

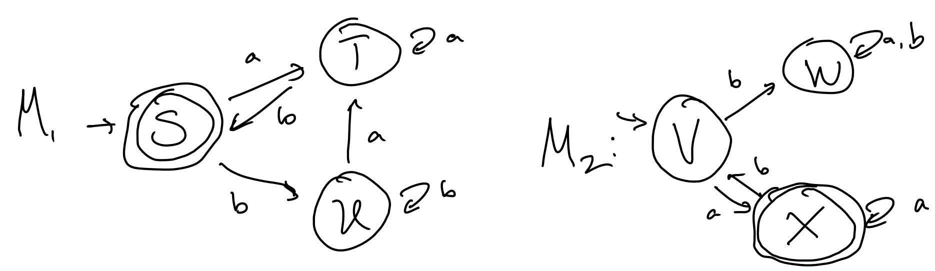
Shown!

Thm Regular languages are closed under intersections.

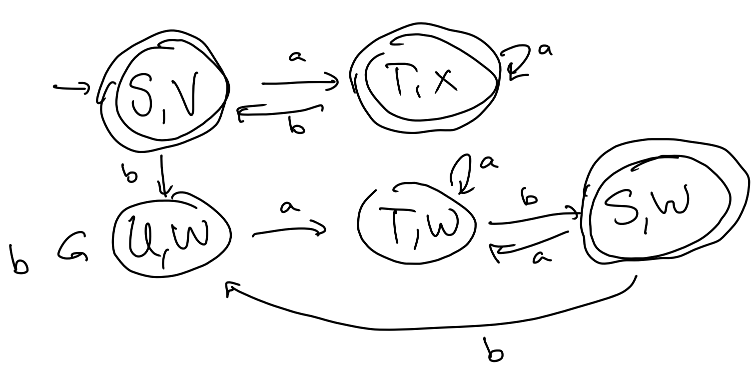
The same basic idea can build a union machine:

Same product construction, but accepting states are pairs (A, T) where

A is accepting OR T is accepting.



A machine for the union:



Accepting any pairs where either one is accepting.

(any with S or X)