$$\frac{d}{da} L = \{ x \mid ax \in L \}$$

removes any initial a from strings in L,

discards any strings in L not starting with a.

$$\frac{d}{da} \{ axb \mid x \in \{a,b\}^* \} = \{ xb \mid x \in \{a,b\}^* \}$$

$$\frac{d}{db} \{ axb \mid x \in \{a,b\}^* \} = \emptyset$$

$$\frac{d}{da} \{ x \mid x \in \{a,b\}^* \} = \{ x \mid ax \in \{a,b\}^* \}$$

ba is in here,

since $a(ba) \in \{a,b\}^*$

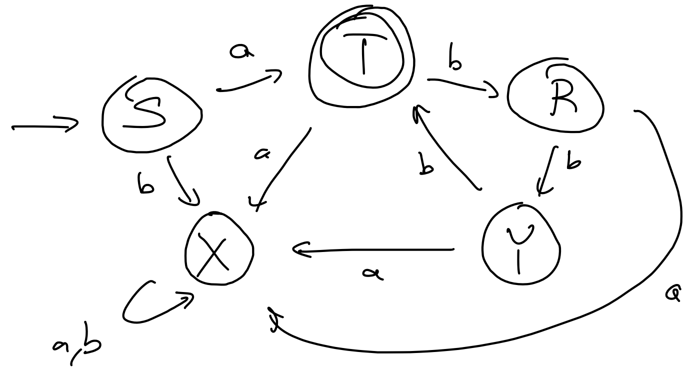
All strings on $\{a,b\}^*$ are in here.

So $$\frac{d}{da} \{ x \mid x \in \{a,b\}^* \} = \{ x \mid x \in \{a,b\}^* \}$$

$$\frac{d}{da} \{ x \mid x \text{ uses an even \# of a's} \} = \{ x \mid x \text{ uses an odd \# of a's} \}$$

$$\frac{d}{db} \{ x \mid x \text{ uses an even \# of a's} \} = \{ x \mid x \text{ uses an even \# of a's} \}$$

M:



$\{a(b^3)^n\}$

Moving the start state to T changes the lang. to $\frac{d}{da}L(M)$

———————— X ———————— $\frac{d}{db}L(M)$

———————— R ———————— $\frac{d}{d(ab)}L(M)$

Note: moving to X makes the language

$\frac{d}{db}L(M)$ and also $\frac{d}{d(aa)}L(M)$.

So $$\frac{d}{db}L(M) = \frac{d}{d(aa)}L(M)$$

similarly: $$\frac{d}{da}L(M) = \frac{d}{d(ab^3)}L(M) = \frac{d}{d(ab^6)}L(M)$$

So many different derivatives will be equal to one of 5 different sets.

5 states

<u>Thm</u> If L is a regular language,
then L has only finitely many derivatives.

<u>Contrapositive:</u> If L has ∞ly many derivatives,
then L is not a regular language.

<u>Favorite ex!</u> $L = \{a^n b^n\}$

$$\frac{d}{da}\{a^n b^n\} = \{a^{n-1} b^n\}$$

$$\frac{d}{da^2}\{\qquad\} = \{a^{n-2} b^n\}$$

$\vdots$ different answer every time,

so L has ∞ly many different derivatives, so L is not regular.