Derivatives for proving a lang is not regular.

Thm If L is regular, then there are only finitely many derivatives.

Contrapos: If L has ∞-ly many derivatives, then L is not regular.

Ex $L = \{a^n b^n\}$ is not regular, since

Let $D_i = \frac{d}{da^i} L = \{a^{n-i} b^n\}$

These are all different, so L has ∞-ly many different derivatives, so L is not regular.

$L = \{a^n b^n c^n\}$ Show it's not regular

Let $D_i = \frac{d}{da^i} L = \{a^{n-i} b^n c^n\}$

these are all different, so L is not regular.

What if L actually is regular?

$$L = \{a^n b^m\} \leftarrow a^* b^*$$

Try the same thing:

$$D_i = \frac{d}{da^i} L = \{a^{n-i} b^m\}$$

These are all equal to $\{a^n b^m\} = L$

$L = \{(ba)^n (ab)^n\}$ is not regular:

<u>Pf</u> Let $D_i = \frac{d}{d(ba)^i} L = \{(ba)^{n-i} (ab)^n\}$

These are all different, so L is not regular.

$$L = \{a b^n a^n\}$$

Let $D_i = \frac{d}{d(ab^i)} L = \{b^{n-i} a^n\}$

These are all different, so L is not regular.

$$L = \{a^m b^n c^n \mid m \in \mathbb{N}, n \in \mathbb{N}\}$$

let $\cancel{D_i = \frac{d}{d(a^m b^i)} L}$ ← can't do that, since m is a variable

Let $D_i = \frac{d}{db^i} L$ ← all strings starting with a get discarded, only those starting with b remain

$= \{ b^{n-i} c^n \}$

These are all different, so L is not regular.

$L = \{ x \mid \#$ of a's in $x = \#$ of b's in $x \}$

Let $D_i = \frac{d}{da^i} L$

$= \{ x \mid \#$ of a's $= (\#$ of b's$) - i \}$

These is all different, so L is not regular.

$L = \{ x \mid \#$ a's $< \#$ b's $\}$

Let $D_i = \frac{d}{da^i} L = \{ \#$ a's $< \#$ b's $- i \}$

These are all different, so L is not regular

$L = \{ x^2 \mid x \in \{a,b\}^* \}$ like $\underline{aab}\,\underline{aab}$

or $\underline{babb}\ \underline{babb}$

Let $D_i = \frac{d}{da^i} L$

$= \{ \underbrace{y a^i y}_{} \mid y \in \{a,b\}^* \}$

we imagine strings like x^2 which begin with a^i

they look like $(\cancel{a^i} y)(a^i y)$

These are all different, so L is not regular.

$L = \{ x x^R \mid x \in \{a,b\}^* \}$ x^R means reversed

$\underbrace{babaa}_{x}\ \underbrace{aabab}_{x^R}$

Let $D_i = \frac{d}{da^i} L = \{ y y^R a^i \mid y \in \{a,b\}^* \}$

Strings in L which begin with a^i look like:

$\underbrace{a^i y}_{x}\ \underbrace{y^R a^i}_{x^R}$ All different!

Show all are not regular:

$$\{b^n a b^n\}$$
$$\{a^n b^m \mid 2n < m\}$$
$$\{x\bar{x} \mid x \in \{0,1\}^*,\ \text{bar means "not"}\}$$

$L = \{b^n a b^n\}$ let $D_i = \frac{d}{db^i} L = \{b^{n-i} a b^n\}$

all different.

$L = \{a^n b^m \mid 2n < m\}$

Let $D_i = \frac{d}{da^i} L = \{a^n b^m \mid 2n < m - i\}$

$2n + i < m$

All different

$\{x\bar{x} \mid x \in \{0,1\}^*\}$

$D_i = \frac{d}{d0^i} L = \{y 1^i \bar{y} \mid y \in \{0,1\}^*\}$ ← all different.

Strings in L which start with 0^i look like: $\underbrace{0^i y}_{x} \underbrace{1^i \bar{y}}_{\bar{x}}$