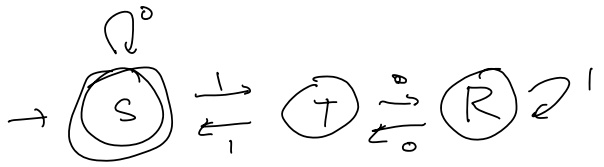


Convert to grammar:

each state becomes a nonterminal,

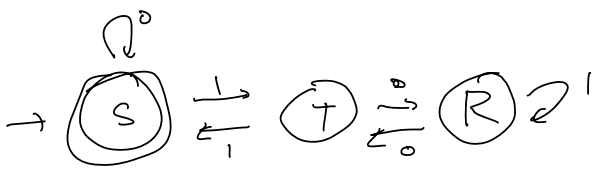
each arrow becomes a grammar production.

each accepting state gets a $\to \varepsilon$ ($\to$ (X) becomes $X \to aY$

$S \to 0S \mid 1T \mid \varepsilon$

$T \to 0R \mid 1S$

$R \to 0T \mid 1R$



01001 is accepted

A grammar derivation should just follow the arrows

$S \Rightarrow 0S \Rightarrow 01T \Rightarrow 010R \Rightarrow 0100T \Rightarrow 01001S$

$\Rightarrow 01001\varepsilon$

$= 01001$

Thm Any regular language can be expressed by a grammar.

do NFA $\longrightarrow$ Grammar.

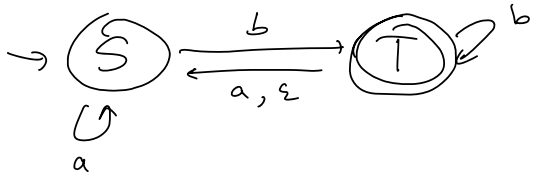
Sometimes we can convert grammar $\to$ NFA

G: $S \to aS \mid bT$

$T \to aS \mid bT \mid S \mid \varepsilon$

εS

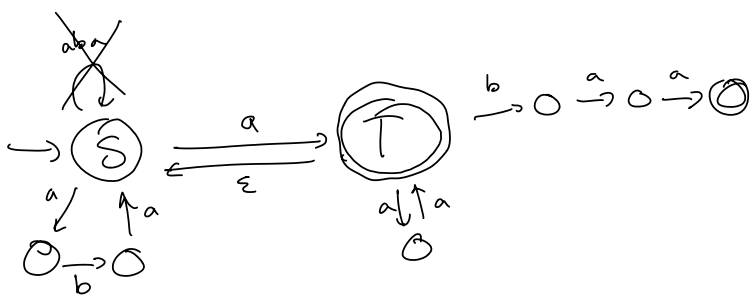
This looks like it came from an NFA.



Slightly fancier:

$S \to abaS \mid aT$

$T \to aaT \mid S \mid \varepsilon \mid baa$

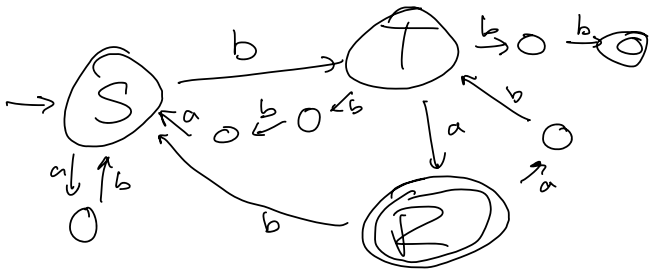


Make an NFA for

$S \to abS \mid bT$

$T \to bbaS \mid aR \mid b^2$

$R \to bS \mid abT \mid \varepsilon$



Context-Free Grammars.

Def G is a <u>context-free grammar</u> (CFG)

when every production looks like

$$X \to s$$

where X is a single nonterminal,

s is any string of terminals & nonterminals.

All our examples have been CFG.

$$S \to aS \mid bT \mid \varepsilon$$
$$T \to Tb^2 \mid bST \mid a^2$$

left side must be a single nonterm.

non-CFG looks like

$$S \to aS \mid bT$$

$T \to abT \mid S$

$aS \to Sa$ ← makes it non-CF.

↑ S can move around, but only in some specific context (on the right of a)

Any language which can be gen. by a CFG is called a context-free language. (CFL)

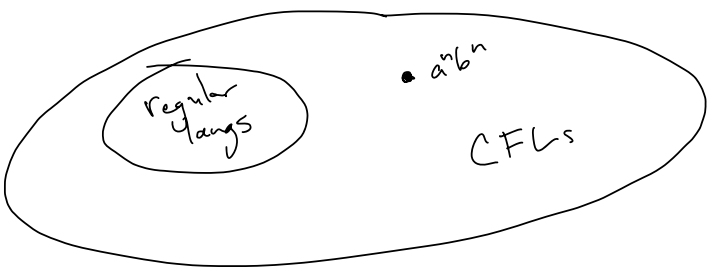
Any NFA can be converted to grammar – actually it'll be CFG.

Thm Any regular lang is a CFL.

Converse is not true:

$G: \quad S \to aSb \mid \varepsilon$

makes $\{a^n b^n\}$ ← non regular, but CFL.



"The Chomsky Heierarchy of languages"

"Make a CFG for this language"

$\{a^n b^n\} \rightarrow S \rightarrow aSb \mid \varepsilon$

Strategies

- If the lang is regular, make a NFA, convert to CFG.

For non-regulars:

Balancing like in $a^n b^n$

each a is "balanced" with 1 b.

Make the grammar so that the a & b are created at the same time.

$S \rightarrow aSb \mid \varepsilon$

~~Don't~~ do:

$S \rightarrow aT \mid \varepsilon$

$T \rightarrow bS$

$a^n b^n$ is $S \rightarrow aSb \mid \varepsilon$

$\{a^n b^{2n}\}$ is: $S \rightarrow aSbb \mid \varepsilon$

each a balances with 2 b's.

$\{a^n b^{n+1}\}$ $\quad S \to aSb \mid b$

each a balances with 1 b,
plus an extra b at the end.

$\{a^m b^n \mid m > n\}$ balanced a's & b's but with extra a's

$$S \to aSb \mid aS \mid a$$

or

$$S \to aSb \mid aA$$
$$A \to aA \mid \varepsilon$$