# Building CFGs

if the lang. is regular:

make a NFA, then convert to grammar.

langs with "balancing" $a^n b^n$

create the a & b together in the same production

$$S \to aSb \mid \varepsilon$$

$\{ b^{2n+2} a^n \}$ ← 2 b's for each a, AND 2 extra b's.

$$S \to bbSa \mid bb$$

Concatenations

$\{ a^n b^n x \mid x$ has an even # of a's $\}$

$a^n b^n$: $S_1 \to aS_1b \mid \varepsilon$

$x$: regular.


$S_2 \to bS_2 \mid aT \mid \varepsilon$

$T \to bT \mid aS_2$

To concatenate two grammars:

$S \to S_1 S_2$

$S_1 \to \ldots$

$S_2 \to \ldots$

$\{ a^n b^n x \mid x$ has an even # of a's $\}$

$S \to S_1 S_2$

$S_1 \to a S_1 b \mid \varepsilon$

$S_2 \to b S_2 \mid aT \mid \varepsilon$

$T \to bT \mid aS_2$

Warning!

don't do inappropriate concats:

$\{ a^n b^n \}$ ← bogus breaking into 2 parts!

~~$S \to S_1 S_2$~~

~~$S_1 \to a S_1 \mid \varepsilon$~~

~~$S_2 \to b S_2 \mid \varepsilon$~~

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Unions: $\{ a^n b^n \} \cup \{ x$ has even # of a's $\}$

same idea, but do $S \to S_1 \mid S_2$

$S_1 \to \ldots$

$S_2 \to \ldots$

$S \to S_1 \mid S_2$

$S_1 \to a S_1 b \mid \varepsilon$

$S_2 \to b S_2 \mid aT \mid \varepsilon$

$T \to bT \mid aS_2$

Ex) Make a CFG for $\{a^n b^m \mid m > n\}$

don't do $\underbrace{a^n}\underbrace{b^m}$

Actually, it's a concatenation like

$\underbrace{a^n b^n}_{S_1}\underbrace{b^k}_{S_2}$ where $k \geq 1$

$S \to S_1 S_2$

$S_1 \to aS_1b \mid \varepsilon$

$S_2 \to bS_2 \mid b$

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$\{a^n b^m \mid m \neq n\}$

$= \overbrace{\{a^n b^m \mid m > n\}}^{S_1} \cup \overbrace{\{a^n b^m \mid n > m\}}^{S_2}$

$S \to S_1 \mid S_2$

$S_1 \to TB$

$T \to aTb \mid \varepsilon$

$B \to bB \mid b$

$S_2 \to AR$

$A \to aA \mid a$

$R \to aRb \mid \varepsilon$

$\{a^n b^n a^m\}$

$\{b^m a^n b^{3m}\}$

$\{x x^R\}$

$\{x \mid x = x^R\}$

$\{x \mid x$ has same # of a's & b's $\}$

$S \to S_1 S_2$

$S_1 \to a S_1 b \mid \varepsilon$

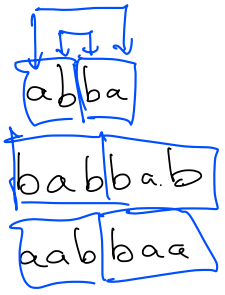
$S_2 \to a S_2 \mid \varepsilon$

$b^m a^n b^{3m}$ each b at beginning matches 3 b's at end; Then extra a's in middle

$S \to bSbbb \mid A$

$A \to aA \mid \varepsilon$

$\{x x^R\}$ like




$S \to aSa \mid bSb \mid \varepsilon$

aabbaa: $S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa$

$\Rightarrow aab\varepsilon baa.$

$\{ x \mid x = x^R \}$ like aabbaa, abbabba

like before, but allows 1 extra letter in middle

$$S \to aSa \mid bSb \mid \varepsilon \mid a \mid b$$