

Stack Machines

	<u>read</u>	<u>pop</u>	<u>push</u>
1.	a	S	SX
2.	b	X	ϵ
3.	ϵ	S	ϵ

Try aabb :

input
aabb
~~a~~abb
~~a~~~~a~~bb
~~a~~~~a~~~~b~~b
~~a~~~~a~~~~b~~~~b~~
~~a~~~~a~~~~b~~~~b~~

Stack

S

SX

SXX

XX

X

ϵ

rule #1

#1

#3

#2

#2

Stack became ϵ ,
so accepted

	<u>read</u>	<u>pop</u>	<u>push</u>
1.	a	S	SX
2.	b	X	ϵ
3.	ϵ	S	ϵ

try abab:

input

abab

~~a~~bab

~~a~~~~b~~ab

~~a~~~~b~~~~a~~b

stack

S

SX

X

ϵ

#1

#3

#2

stack! s, abab
is rejected.

	<u>read</u>	<u>pop</u>	<u>push</u>
1.	a	S	SX
2.	b	X	ϵ
3.	ϵ	S	ϵ

input remaining to be read
↓
stack

aabb: $(aabb, S) \mapsto_1 (abb, SX) \mapsto_1 (bb, SXX)$
 $\mapsto_3 (bb, XX) \mapsto_2 (b, X) \mapsto_2 (\epsilon, \epsilon)$

accepted!

$(abab, S) \mapsto_1 (bab, SX) \mapsto_3 (bab, X) \mapsto_2 (ab, \epsilon)$
 rejected.

The pairs are called Instantaneous Descriptions (IDs)

This stack machine has language $\{a^n b^n\}$

	<u>read</u>	<u>pop</u>	<u>push</u>
1	a	S	X
2	b	X	Y
3	b	Y	S
4	ϵ	S	ϵ

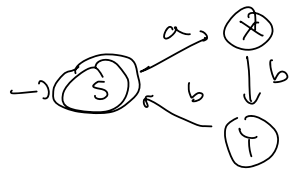
try abb

$(abb, S) \mapsto_1 (bb, X) \mapsto_2 (b, Y)$

$\mapsto_3 (\epsilon, S) \mapsto_4 (\epsilon, \epsilon)$ accepted!

$(aab, S) \mapsto_1 (ab, X)$ rejected.
 stuck!

Stack is only ever S, X, Y (or ϵ)
 there are only 3 possible states.



This NFA is equivalent to the stack machine.

$$L = \{ (abb)^n \}$$

This shows how to convert NFA to stack mach.

So any regular language can be computed by stack machine.

A language for balanced brackets:

$$\Sigma = \{ [,] \}$$

The Dyck language all strings of $[$ & $]$ which are properly balanced.

accepts: $[[[]][[]][[]]]$

rejects: $] [$ or $[]]]$

or $[[[]][[]]] []$

Stack idea: when I see $[$, push X onto the stack.

when I see $]$, pop X from the stack

read	pop	push
[	ϵ	X
]	X	ϵ
ϵ	S	ϵ

$$([[] [[]]], S) \mapsto ([] [[]] , XS)$$

$$\mapsto ([] [[]] , XXS) \mapsto ([[]] , XS)$$

$$\mapsto ([[]] , XXS) \mapsto ([]] , XXXS)$$

$$\mapsto ([] , XXS) \mapsto ([] , XS) \mapsto (\epsilon, S) \mapsto (\epsilon, \epsilon)!$$

accepted

$$([] , S) \mapsto ([] , XS) \mapsto ([] , S) \mapsto ([] , \epsilon)$$

stuck! rejected.