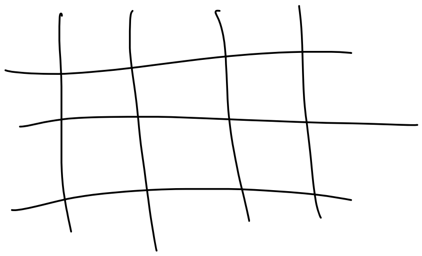
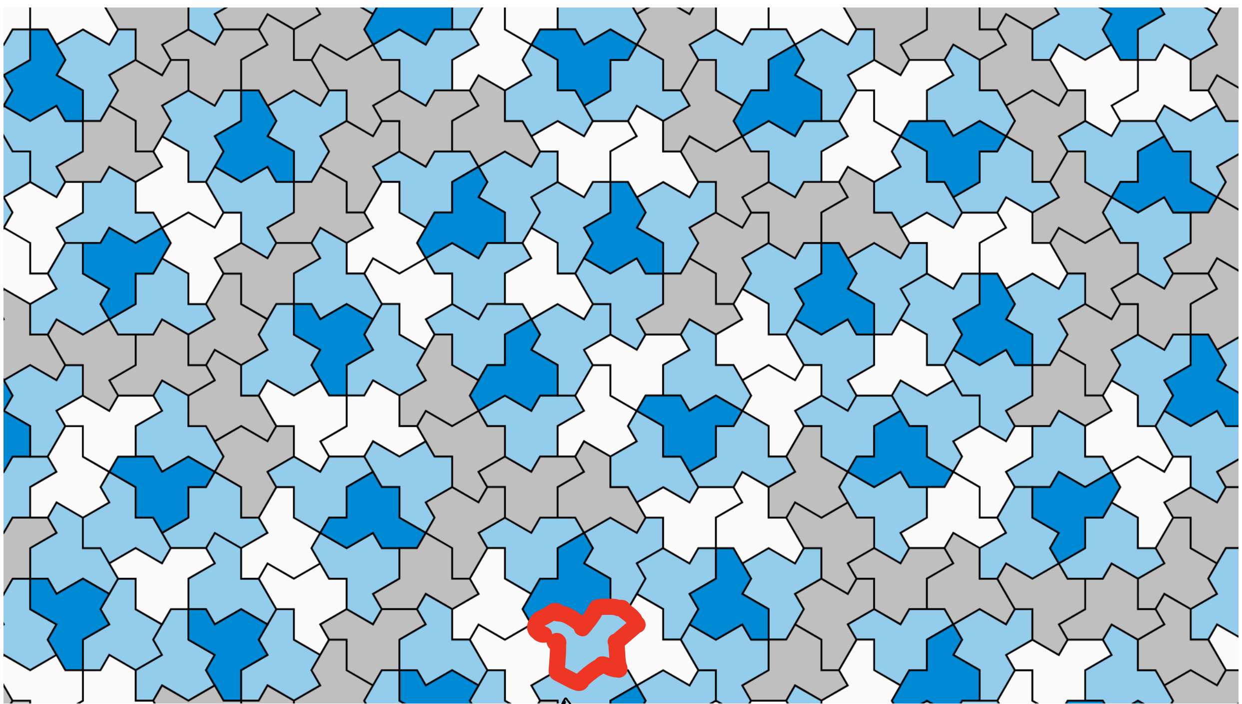
Aperiodic Tilings "Penrose tiling"



is periodic



aperiodic monotile

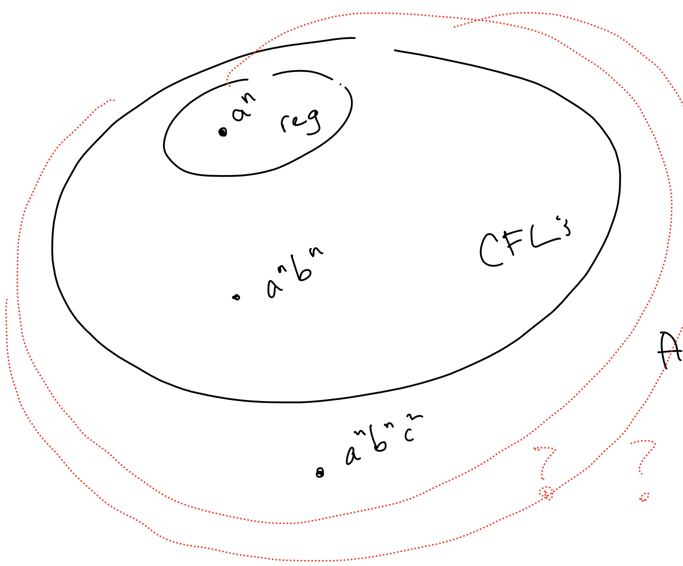
Stack machines vs CFG

We can convert stack $\to$ CFG

and CFG $\to$ stack

So Stack machines & CFG

have the same "expressive power"



Are there interesting non-CF languages?

Are there machines for them?

The standard non-CF language: $a^n b^n c^n$

$a^n b^n$ is $S \to aSb \mid \varepsilon$

try $a^n b^n c^n$: ~~$S \to aSbc \mid \varepsilon$ makes $a^n (bc)^n$~~

$$S \to aSbSc \mid \varepsilon$$

$$S \Rightarrow aSbSc \Rightarrow a\,aSbSc\,b\,aSbSc\,c \Rightarrow aabcbabcc$$

$a^nb^nc^n$:

$$S \to aSBc \mid abc \mid \varepsilon$$

not CF!

$$\begin{cases} cB \to Bc \\ bB \to bb \end{cases}$$

B turns into b, but must move left through all c's first to join with the other b's.

Actually any lang. with more than 2 balanced exponents is non-CF.

Real world prog-langs with indentation:

```
XXXX                 TT xxx
T XXXX               TT xxx
T XXXX       or      TT xxxx
T XXXX
```

TT xx
TT xx
T xxx
xxxx

Blocks using the same indent level look like:

$T^n xxxx\ T^n xxx\ T^n xxxx$

So a lang with meaningful indents is never CF.

Python is not CF.

Turing Machines

in early 1900s, axioms were a big deal in math.

1880's – axioms in algebra, linear algebra
1890s – Hilbert geometry
1900s – axioms for set theory (ZFC)

All of math should be expressible using finitely many axioms.

Can all math. questions be either proved or disproved from the axioms?

NO

Gödel

Turing's question: Is there a single master $\boxed{\text{procedure}}$ for proving all theorems from the axioms?

What counts as a mathematical procedure?

an algorithm

Alonzo Church: any recursive combination of "λ-terms"

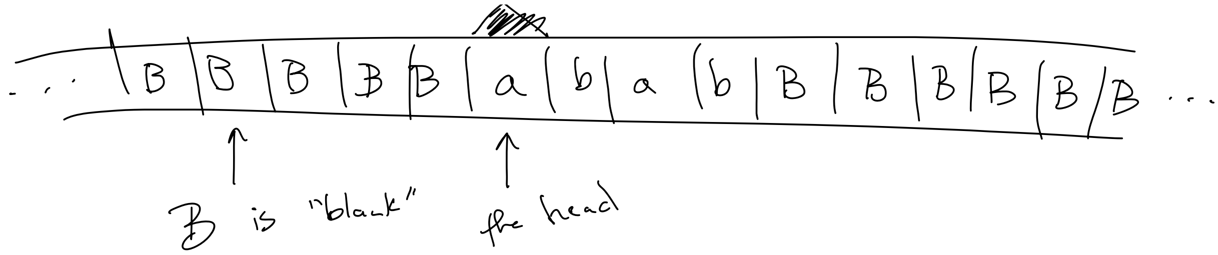
Alan Turing: a machine.

T. described his machine's capabities "A universal machine"

The Turing Machine (TM)

Like a DFA with states & arrows, also memory like a stack, The Tape

The tape is ∞-long, it has a moving "head" which can read & write symbols on the tape.



In each step, the head reads the symbol, changes it (or leaves it), then moves one step L or R

If the TM ever enters an accepting state in its state diagram, then it stops and accepts.

We begin the TM in the starting state, with the input written on the tape, head pointing at the first letter.