A function $f : \Sigma^* \to \Sigma^*$ is Turing Computable

if $\exists$ a TM which performs f

on the tape.

(i.e. tape starts with x,
finishes with $f(x)$)

like $f(x) = ax$ or $f(x) = abaxbb$

Sometimes our domain isn't all of Σ^*,

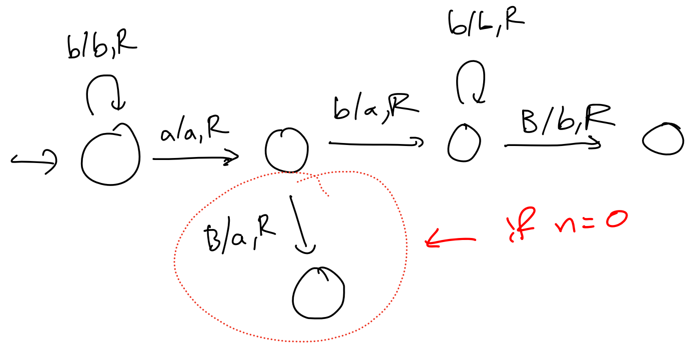
often we consider $f : S \to \Sigma^*$

where $S \subseteq \Sigma^*$

Ex $S = \{ b^n a b^n \}$

$f(b^n a b^n) = b^n a a b^n$

change 1st b
to a,
then add b at the end.



$f(xcx) = xx$ for $x \in \{a,b\}^*$

abbb c abbb

↓

abbbabbb

abbb

↓

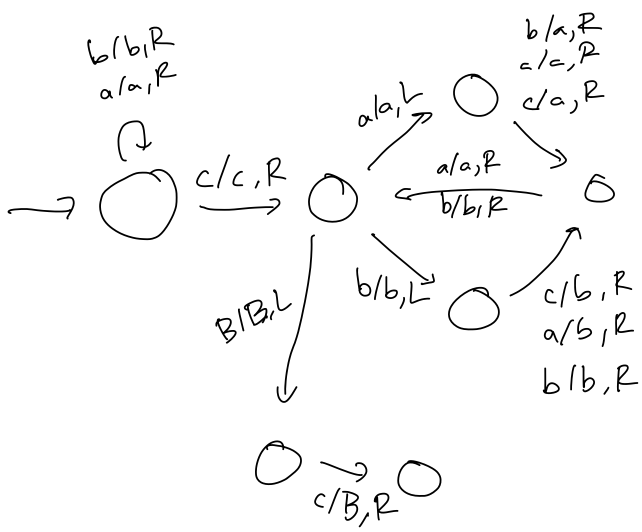
$\bar{a}$bbbabbb

$\bar{a}$bbba

after the c:

copy letters 1 by 1,

one spot to the left.



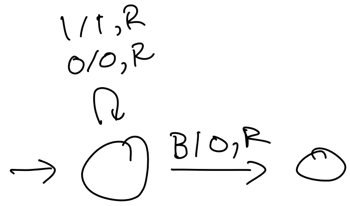
Simple numerical functions:

If x is a binary number like 1000 (8 in decimal)

$f(x) = 2x$

10101101 → 1010111010

add a 0 digit to the end.



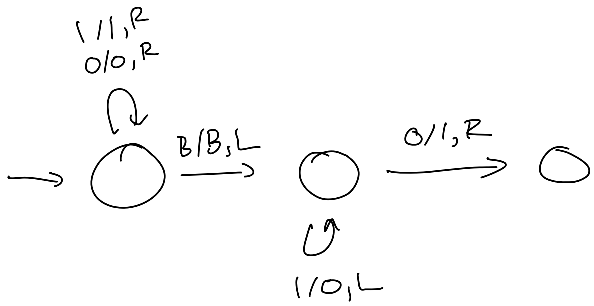
$f(x) = x + 1$

if $x = 1101010$

then $x+1 = 1101011$

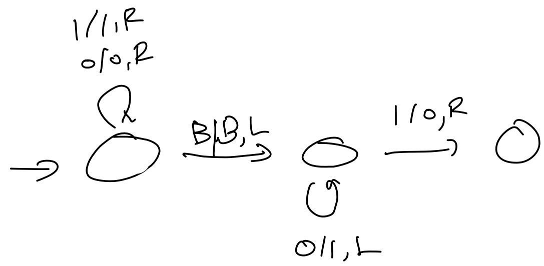
if $x = 1100111$

$$\begin{array}{r} 1100111 \\ +\quad 1 \\ \hline 1101000 \end{array}$$



Try $f(x) = x - 1$ (assume $x \geq 1$)

$$\begin{array}{r} 100101 \\ -\quad 1 \\ \hline 100100 \end{array} \qquad \begin{array}{r} 100100 \\ -\quad 1 \\ \hline 100\underbrace{011} \end{array}$$



$\boxed{1}$

if $x = 0$

this makes ...111111