

Uncomputable things

L_h : Given M & x , decide whether or not M halts on x
"The Halting Problem"

L_u : decide whether or not M accepts x .

Σ ← halting probability

More ordinary uncomputable problems:

"Diophantine Equations"

Multivariable Polynomials:

$$3x^2 + 4y - 8xy = 5x + 1$$

can we solve it in integers?

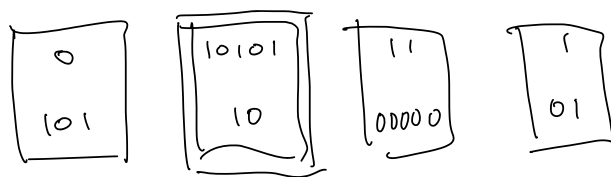
(or say no solutions exist)

in 1970, proved to be uncomputable.

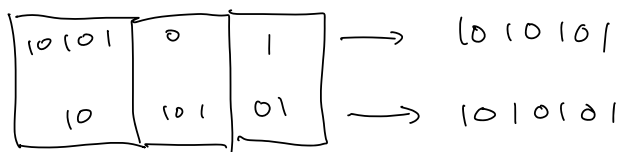
↑
shown to be equiv to
the Halting Problem.

The Post Correspondence Problem (1940s) (PCP)

Given blocks like this:



Can we arrange them (repeats allowed), starting with the starting tile, so that the string read across the top equals the string across the bottom.



$\rightarrow 1010101$

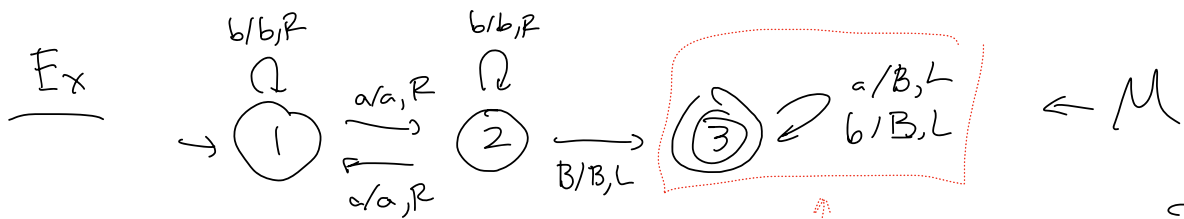
$\rightarrow 1010101$

PCP is undecidable by TM.

We'll show PCP is equivalent to L_u

$M \# x$ where M accepts x .

We can encode any TM as a set of PCP tiles.



with input $x = ab$. Is ab accepted?

the TM "clears the tape" when it accepts.

is $x=ab$ accepted?

$1ab \mapsto a2b \mapsto ab2 \mapsto a3b \mapsto 3a \mapsto 3 \rightarrow$

Translate this into PCP files

letters:

a
a

b
b

$\mapsto$
$\mapsto$

move R:

1a
a2

1b
b1

2b
b2

2a
a1

move L:

a2
3a

b2
3b

2
3

a3a
3a

b3a
3b

3a
3

a3b
3a

b3b
3b

3b
3

accept state:

$3 \mapsto$
ϵ

start state:

ϵ
1ab $\mapsto$

Solving the PCP is the same as a successful derivation!

ϵ
1ab $\mapsto$

1a
a2

b
b

$\mapsto$
$\mapsto$

a
a

2b
b2

$\mapsto$
$\mapsto$

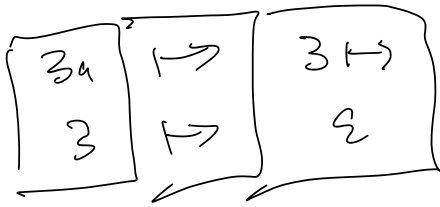
a
a

b2
3b

$\mapsto$
$\mapsto$

a3b
3a

$\mapsto$
$\mapsto$



So any PCP solver could
be used as an oracle for LU.

So PCP is not computable by
any TM.