Math 3385

HW #1

#1, 3, 8, 9

#1 $X = \{a, b\}$

possible topologies: $\{\emptyset, \{a,b\}\}$

$\{\emptyset, \{a\}, \{a,b\}\}$

$\{\emptyset, \{b\}, \{a,b\}\}$

$\{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

#3 $\mathcal{T}$ is the discrete top iff $\{x\} \in \mathcal{T}$ $\forall x \in X$.

$\Rightarrow$ Assume $\mathcal{T}$ is the discrete topology.
then all sets are open, so $\{x\}$ is open $\forall x$.

$\Leftarrow$ Assume $\{x\}$ is open $\forall x$, let $\mathcal{U} \subset X$, will show $\mathcal{U}$ is open.

$\mathcal{U} = \bigcup_{x \in \mathcal{U}} \{x\}$, so $\mathcal{U}$ is a union of open sets,

so $\mathcal{U}$ is open.

#8

i $\emptyset$ and X are in $\mathcal{T}$ by definition

ii Let $U_1, \dots, U_n$ be open WTS $\bigcap_{i=1}^{n} U_i$ is open.

Since U_i is open, $p \notin U_i \ \forall i$.

Thus $p \notin \bigcap_{i=1}^{n} U_i$, so $\bigcap_{i=1}^{n} U_i$ is open as desired.

iii Let U_α be open $\forall \alpha \in A$, so $p \notin U_\alpha$.

Then $p \notin \bigcup_{\alpha \in A} U_\alpha$, so $\bigcup_{\alpha \in A} U_\alpha$ is open as desired.

#9

i $\emptyset$ & $\mathbb{R}$ are open by definition.

ii Let $U_1, \dots, U_n$ be open.

So $U_i = (-\infty, p_i)$ for each i,

and $\bigcap_{i=1}^{n} U_i = (-\infty, \min(p_i))$,

so $\bigcap_{i=1}^{n} U_i$ is open.

iii Let U_α be open,

say $U_\alpha = (-\infty, p_\alpha)$ for each α.

Then $\bigcup_{\alpha \in A} U_\alpha = (-\infty, \sup p_\alpha)$

or $= \mathbb{R}$ if the p_α are unbounded.

Either way, $\bigcup_{\alpha \in A} U_\alpha$ is open.