Math 3385
Homework #2

Section 1.2 #19a
Section 1.3 #27a
Section 2.1 #1h&i, 2c

1.2 #19a

Vertical intervals top

$\mathcal{B} = \{ \{a\} \times (b,c) \mid a,b,c \in \mathbb{R} \}$ B a basis.

i (covering) Take any $(x,y) \in \mathbb{R}^2$,

then $(x,y) \in \{x\} \times (y-1, y+1)$

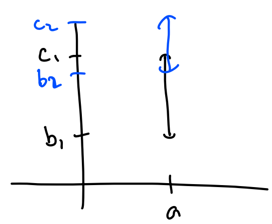
so (x,y) is in some basis set.

ii Let $B_1, B_2 \in \mathcal{B}$, say

$B_1 = \{a_1\} \times (b_1, c_1)$

$B_2 = \{a_2\} \times (b_2, c_2)$,

assume $x \in B_1 \cap B_2$. Then these vertical intervals overlap, so $a_1 = a_2$, and we can assume they look like:



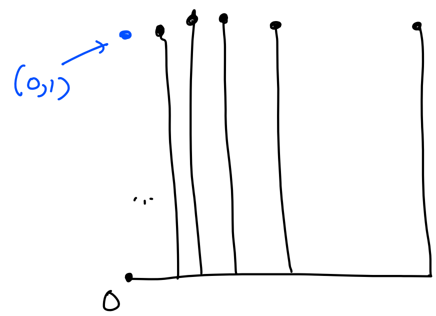
Assuming $b_1 < c_1 < b_2 < c_2$,

we can let $B_3 = \{a\} \times (b_2, c_2)$,

and $B_3 \subset B_1 \cap B_2$ as desired.

1.3 #27a

C is not closed - enough to show $\mathbb{R}^2 - C$ is not open



Consider $(0,1) \in \mathbb{R}^2 - C$.

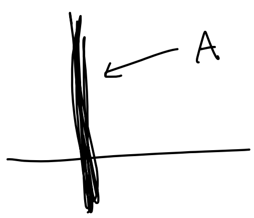
Any nbhd of $(0,1)$ will contain points of C,

so $(0,1)$ has no nbhd contained in $\mathbb{R}^2 - C$,

so $\mathbb{R}^2 - C$ is not open,

so C is not closed.

2.1 #1h



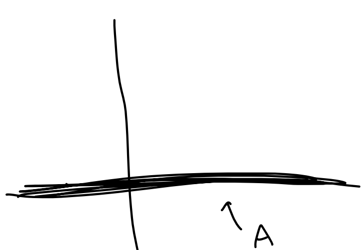
With vert-interval top, A is an open set, so

$\mathring{A} = A$.

Also A is closed, since the complement is open.

So $\overline{A} = A$.

#1 i



A contains no vertical intervals,

so $\mathring{A} = \emptyset$,

A is closed, since the complement is open. So $\overline{A} = A$.

2.1 #2c

A is closed iff $A = Cl(A)$

$\Leftarrow$ Assuming $A = Cl(A)$, then A is closed since $Cl(A)$ is closed.

$\Rightarrow$ Assume A is closed.

Then the smallest closed set containing A is A!

So $A = Cl(A)$.